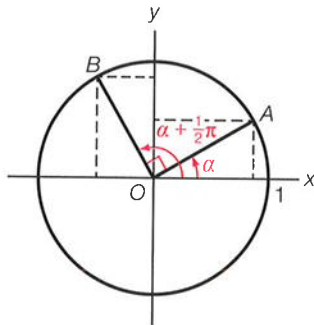


12 Goniometrische formules

Voorkennis Goniometrische formules herleiden

Bladzijde 143

1 a

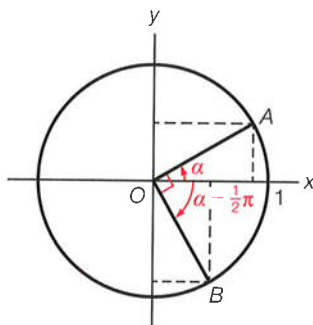


$$\sin(\alpha + \frac{1}{2}\pi) = y_B = x_A = \cos(\alpha) \text{ oftewel } \sin(\alpha + \frac{1}{2}\pi) = \cos(\alpha).$$

$$\cos(\alpha + \frac{1}{2}\pi) = x_B = -y_A = -\sin(\alpha) \text{ oftewel } \cos(\alpha + \frac{1}{2}\pi) = -\sin(\alpha).$$

b $\sin(\alpha) = -\cos(\alpha + \frac{1}{2}\pi) = \cos(\alpha + \frac{1}{2}\pi + \pi) = \cos(\alpha + 1\frac{1}{2}\pi) = \cos(\alpha + 1\frac{1}{2}\pi - 2\pi) = \cos(\alpha - \frac{1}{2}\pi)$

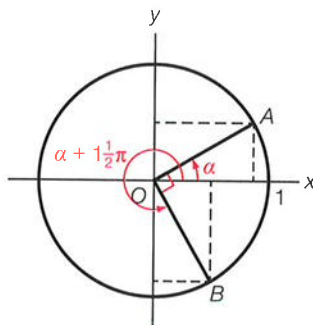
2 a



$$y = \sin(\alpha - \frac{1}{2}\pi) = y_B = -x_A = -\cos(\alpha), \text{ dus } y = -\cos(\alpha).$$

b $y = \cos(\alpha - \frac{1}{2}\pi) = x_B = y_A = \sin(\alpha), \text{ dus } y = \sin(\alpha).$

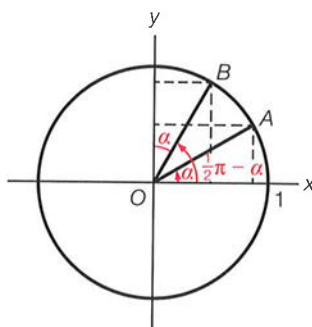
c



$$y = \sin(\alpha + 1\frac{1}{2}\pi) = y_B = -x_A = -\cos(\alpha), \text{ dus } y = -\cos(\alpha).$$

d $y = \cos(\alpha + 1\frac{1}{2}\pi) = x_B = y_A = \sin(\alpha), \text{ dus } y = \sin(\alpha).$

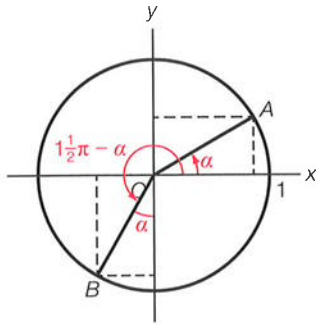
e



$$y = \sin(\frac{1}{2}\pi - \alpha) = y_B = x_A = \cos(\alpha), \text{ dus } y = \cos(\alpha).$$

f $y = \cos(\frac{1}{2}\pi - \alpha) = x_B = y_A = \sin(\alpha)$, dus $y = \sin(\alpha)$.

g



$y = \sin(1\frac{1}{2}\pi - \alpha) = y_B = -x_A = -\cos(\alpha)$, dus $y = -\cos(\alpha)$.

h $y = \cos(1\frac{1}{2}\pi - \alpha) = x_B = -y_A = -\sin(\alpha)$, dus $y = -\sin(\alpha)$.

Bladzijde 144

- 3 a $y = \sin(2x - \frac{1}{3}\pi) = \cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(2x - \frac{5}{6}\pi)$
 b $y = -\sin(3x - \frac{1}{4}\pi) = \sin(3x - \frac{1}{4}\pi + \pi) = \sin(3x + \frac{3}{4}\pi) = \cos(3x + \frac{3}{4}\pi - \frac{1}{2}\pi) = \cos(3x + \frac{1}{4}\pi)$
 c $y = \cos(\frac{1}{2}x + \frac{1}{4}\pi) = \sin(\frac{1}{2}x + \frac{1}{4}\pi + \frac{1}{2}\pi) = \sin(\frac{1}{2}x + \frac{3}{4}\pi)$
 d $y = -\cos(2(x - \frac{1}{6}\pi)) = \cos(2x - \frac{1}{3}\pi + \pi) = \cos(2x + \frac{2}{3}\pi) = \sin(2x + \frac{2}{3}\pi + \frac{1}{2}\pi) = \sin(2x + 1\frac{1}{6}\pi)$

12.1 Goniometrische formules bij vergelijkingen en herleidingen

Bladzijde 145

- 1 a $\sin(2x) + \sin(x) = 0$
 $\sin(2x) = -\sin(x)$
 $\sin(2x) = \sin(x + \pi)$
 b $2\sin^2(x) = 3\cos(x)$
 $2(1 - \cos^2(x)) = 3\cos(x)$
 $2 - 2\cos^2(x) = 3\cos(x)$
 $2\cos^2(x) + 3\cos(x) - 2 = 0$

Bladzijde 147

- 2 $\sin(2x - \frac{1}{3}\pi) = -\cos(x + \frac{1}{3}\pi)$
 $\cos(2x - \frac{5}{6}\pi) = \cos(x + 1\frac{1}{3}\pi)$
 $2x - \frac{5}{6}\pi = x + 1\frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -x - 1\frac{1}{3}\pi + k \cdot 2\pi$
 $x = 2\frac{1}{6}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = 2\frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$
- 3 a $\sin(x + \frac{1}{2}\pi) = \cos(2x)$
 $\cos(x) = \cos(2x)$
 $x = 2x + k \cdot 2\pi \vee x = -2x + k \cdot 2\pi$
 $-x = k \cdot 2\pi \vee 3x = k \cdot 2\pi$
 $x = k \cdot 2\pi \vee x = k \cdot \frac{2}{3}\pi$
 x in $[0, 2\pi]$ geeft $x = 0 \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = 2\pi$
 b $\sin(3x) = -\cos(x)$
 $\cos(3x - \frac{1}{2}\pi) = \cos(x + \pi)$
 $3x - \frac{1}{2}\pi = x + \pi + k \cdot 2\pi \vee 3x - \frac{1}{2}\pi = -(x + \pi) + k \cdot 2\pi$
 $2x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee 3x - \frac{1}{2}\pi = -x - \pi + k \cdot 2\pi$
 $x = \frac{3}{4}\pi + k \cdot \pi \vee 4x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = \frac{3}{4}\pi + k \cdot \pi \vee x = -\frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{3}{4}\pi \vee x = 1\frac{3}{4}\pi \vee x = \frac{3}{8}\pi \vee x = \frac{7}{8}\pi \vee x = 1\frac{3}{8}\pi \vee x = 1\frac{7}{8}\pi$

c $\sin^2(x) + \frac{1}{2}\cos(x) = 1$
 $1 - \cos^2(x) + \frac{1}{2}\cos(x) - 1 = 0$
 $-\cos^2(x) + \frac{1}{2}\cos(x) = 0$
 $-\cos(x)(\cos(x) - \frac{1}{2}) = 0$
 $\cos(x) = 0 \vee \cos(x) = \frac{1}{2}$
 $x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = \frac{1}{3}\pi \vee x = \frac{5}{3}\pi$

d $\sin(\frac{1}{2}x) + \sqrt{3} \cdot \cos(\frac{1}{2}x) = 0$
 $\sin(\frac{1}{2}x) = -\sqrt{3} \cdot \cos(\frac{1}{2}x)$
 $\frac{\sin(\frac{1}{2}x)}{\cos(\frac{1}{2}x)} = -\sqrt{3}$
 $\tan(\frac{1}{2}x) = -\sqrt{3}$
 $\frac{1}{2}x = -\frac{1}{3}\pi + k \cdot \pi$
 $x = -\frac{2}{3}\pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{4}{3}\pi$

4 a $\cos(2\pi t) = \sin(\frac{1}{2}\pi t)$
 $\cos(2\pi t) = \cos(\frac{1}{2}\pi t - \frac{1}{2}\pi)$
 $2\pi t = \frac{1}{2}\pi t - \frac{1}{2}\pi + k \cdot 2\pi \vee 2\pi t = -(\frac{1}{2}\pi t - \frac{1}{2}\pi) + k \cdot 2\pi$
 $\frac{1}{2}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi \vee 2\pi t = -\frac{1}{2}\pi t + \frac{1}{2}\pi + k \cdot 2\pi$
 $\frac{1}{2}t = -\frac{1}{2} + k \cdot 2 \vee 2\frac{1}{2}t = \frac{1}{2} + k \cdot 2\pi$
 $t = -\frac{1}{3} + k \cdot \frac{1}{3} \vee 2\frac{1}{2}t = \frac{1}{2} + k \cdot 2$
 $t = -\frac{1}{3} + k \cdot \frac{1}{3} \vee t = \frac{1}{5} + k \cdot \frac{4}{5}$
 t in $[0, 3]$ geeft $t = 1 \vee t = 2\frac{1}{3} \vee t = \frac{1}{5} \vee t = 1\frac{4}{5} \vee t = 2\frac{3}{5}$

b $\sin(\frac{1}{6}\pi t) = -\cos(\pi t)$
 $\cos(\frac{1}{6}\pi t - \frac{1}{2}\pi) = \cos(\pi t + \pi)$
 $\frac{1}{6}\pi t - \frac{1}{2}\pi = \pi t + \pi + k \cdot 2\pi \vee \frac{1}{6}\pi t - \frac{1}{2}\pi = -\pi t - \pi + k \cdot 2\pi$
 $\frac{1}{6}t - \frac{1}{2} = t + 1 + k \cdot 2 \vee \frac{1}{6}t - \frac{1}{2} = -t - 1 + k \cdot 2$
 $-\frac{5}{6}t = 1\frac{1}{2} + k \cdot 2 \vee \frac{1}{6}t = -\frac{1}{2} + k \cdot 2$
 $t = -1\frac{4}{5} + k \cdot 2\frac{2}{5} \vee t = -\frac{3}{7} + k \cdot \frac{5}{7}$
 t in $[0, 3]$ geeft $t = \frac{3}{5} \vee t = 3 \vee t = 1\frac{2}{7}$

5 a $2\sin^2(x) + \cos^2(x) + \cos(x) = 0$
 $2(1 - \cos^2(x)) + \cos^2(x) + \cos(x) = 0$
 $2 - 2\cos^2(x) + \cos^2(x) + \cos(x) = 0$
 $\cos^2(x) - \cos(x) - 2 = 0$
 $(\cos(x) + 1)(\cos(x) - 2) = 0$
 $\cos(x) = -1 \vee \cos(x) = 2$
 $x = \pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = \pi$

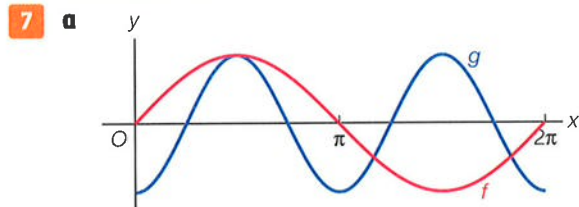
b $\sin(2x - \frac{1}{3}\pi) = -\cos(\frac{1}{2}x)$
 $\cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(\frac{1}{2}x + \pi)$
 $\cos(2x - \frac{5}{6}\pi) = \cos(\frac{1}{2}x + \pi)$
 $2x - \frac{5}{6}\pi = \frac{1}{2}x + \pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -\frac{1}{2}x - \pi + k \cdot 2\pi$
 $\frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{5}{3}\pi + k \cdot 4\pi \vee x = -\frac{1}{3}\pi + k \cdot 4\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{2}{3}\pi \vee x = \frac{11}{3}\pi \vee x = \frac{8}{3}\pi$

c $\cos(\frac{1}{4}\pi t) = -\sin(\frac{2}{5}\pi t)$
 $\sin(\frac{1}{4}\pi t + \frac{1}{2}\pi) = \sin(\frac{2}{5}\pi t + \pi)$
 $\frac{1}{4}\pi t + \frac{1}{2}\pi = \frac{2}{5}\pi t + \pi + k \cdot 2\pi \vee \frac{1}{4}\pi t + \frac{1}{2}\pi = \pi - (\frac{2}{5}\pi t + \pi) + k \cdot 2\pi$
 $\frac{1}{4}t + \frac{1}{2} = \frac{2}{5}t + 1 + k \cdot 2 \vee \frac{1}{4}t + \frac{1}{2} = 1 - (\frac{2}{5}t + 1) + k \cdot 2$
 $-\frac{3}{20}t = \frac{1}{2} + k \cdot 2 \vee \frac{13}{20}t = -\frac{1}{2} + k \cdot 2$
 $t = -\frac{10}{3} + k \cdot \frac{40}{3} \vee t = -\frac{10}{13} + k \cdot \frac{40}{13}$
 t in $[0, 10]$ geeft $t = 10 \vee t = 2\frac{4}{13} \vee t = 5\frac{5}{13} \vee t = 8\frac{6}{13}$

d $\cos(\frac{1}{3}\pi x) = \sqrt{3} \cdot \sin(\frac{1}{3}\pi x)$
 $\frac{\sin(\frac{1}{3}\pi x)}{\cos(\frac{1}{3}\pi x)} = \frac{1}{\sqrt{3}}$
 $\tan(\frac{1}{3}\pi x) = \frac{1}{\sqrt{3}}$
 $\frac{1}{3}\pi x = \frac{1}{6}\pi + k \cdot \pi$
 $\frac{1}{3}x = \frac{1}{6} + k \cdot 1$
 $x = \frac{1}{2} + k \cdot 3$
 x in $[0, 10]$ geeft $x = \frac{1}{2} \vee x = 3\frac{1}{2} \vee x = 6\frac{1}{2} \vee x = 9\frac{1}{2}$

6 a $2\sin(x) = \sin(x)$
 $\sin(x) = 0$
b $\sin(2x) = \sin(x)$
 $2x = x + k \cdot 2\pi \vee 2x = \pi - x + k \cdot 2\pi$
c —
d —
e $\sin(2x) = \sin(x + \frac{1}{3}\pi)$
 $2x = x + \frac{1}{3}\pi + k \cdot 2\pi \vee 2x = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$
f —

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b $f(x) = -\frac{1}{2}\sqrt{2}$ geeft $\sin(x) = -\frac{1}{2}\sqrt{2}$
 $x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = \pi - \frac{1}{4}\pi + k \cdot 2\pi$
 $x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = 1\frac{1}{4}\pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = 1\frac{1}{4}\pi \vee x = 1\frac{3}{4}\pi$
c $g(x) = \frac{1}{2}$ geeft $-\cos(2x) = \frac{1}{2}$
 $\cos(2x) = -\frac{1}{2}$
 $2x = \frac{2}{3}\pi + k \cdot 2\pi \vee 2x = -\frac{2}{3}\pi + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + k \cdot \pi \vee x = -\frac{1}{3}\pi + k \cdot \pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = \frac{2}{3}\pi \vee x = 1\frac{2}{3}\pi$
d $f(x) = g(x)$ geeft $\sin(x) = -\cos(2x)$
 $\cos(x - \frac{1}{2}\pi) = \cos(2x + \pi)$
 $x - \frac{1}{2}\pi = 2x + \pi + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -(2x + \pi) + k \cdot 2\pi$
 $-x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -2x - \pi + k \cdot 2\pi$
 $x = -1\frac{1}{2}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = -1\frac{1}{2}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$
 $f(x) \leq g(x)$ geeft $x = \frac{1}{2}\pi \vee 1\frac{1}{6}\pi \leq x \leq 1\frac{5}{6}\pi$

8 $f(x) = g(x)$ geeft $\sin(2x - \frac{1}{3}\pi) = -\cos(x - \frac{1}{6}\pi)$
 $\cos(2x - \frac{5}{6}\pi) = \cos(x + \frac{5}{6}\pi)$
 $2x - \frac{5}{6}\pi = x + \frac{5}{6}\pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -x - \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{5}{3}\pi + k \cdot 2\pi \vee 3x = k \cdot 2\pi$
 $x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = k \cdot \frac{2}{3}\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{2}{3}\pi \vee x = 0 \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = \pi$

$$O(V) = \int_0^{\frac{2}{3}\pi} (f(x) - g(x)) dx = \int_0^{\frac{2}{3}\pi} (\sin(2x - \frac{1}{3}\pi) + \cos(x - \frac{1}{6}\pi)) dx = \left[-\frac{1}{2}\cos(2x - \frac{1}{3}\pi) + \sin(x - \frac{1}{6}\pi)\right]_0^{\frac{2}{3}\pi}$$

$$= -\frac{1}{2}\cos(\pi) + \sin(\frac{1}{2}\pi) - (-\frac{1}{2}\cos(-\frac{1}{3}\pi) + \sin(-\frac{1}{6}\pi)) = \frac{1}{2} + 1 - (-\frac{1}{4} - \frac{1}{2}) = 2\frac{1}{4}$$

$$O(W) = \int_{\frac{1}{3}\pi}^{\frac{4}{3}\pi} (f(x) - g(x)) dx = \int_{\frac{1}{3}\pi}^{\frac{4}{3}\pi} (\sin(2x - \frac{1}{3}\pi) + \cos(x - \frac{1}{6}\pi)) dx = \left[-\frac{1}{2}\cos(2x - \frac{1}{3}\pi) + \sin(x - \frac{1}{6}\pi)\right]_{\frac{1}{3}\pi}^{\frac{4}{3}\pi}$$

$$= -\frac{1}{2}\cos(3\pi) + \sin(1\frac{1}{2}\pi) - (-\frac{1}{2}\cos(2\frac{1}{3}\pi) + \sin(1\frac{1}{6}\pi)) = \frac{1}{2} - 1 - (-\frac{1}{4} - \frac{1}{2}) = \frac{1}{4}$$

De oppervlakte van V is $\frac{2\frac{1}{4}}{\frac{1}{4}} = 9$ keer zo groot als de oppervlakte van W .

9 $f(x) = \frac{1}{2}$ geeft $\sin(2x - \frac{1}{3}\pi) = \frac{1}{2}$
 $2x - \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi$
 $2x = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x = 1\frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{4}\pi + k \cdot \pi \vee x = \frac{7}{12}\pi + k \cdot \pi$

Het is duidelijk dat het kleinste deel het deel boven de lijn $y = \frac{1}{2}$ is.

$$\text{Dus } O = \int_{\frac{1}{4}\pi}^{\frac{7}{12}\pi} (f(x) - \frac{1}{2}) dx = \int_{\frac{1}{4}\pi}^{\frac{7}{12}\pi} (\sin(2x - \frac{1}{3}\pi) - \frac{1}{2}) dx = \left[-\frac{1}{2}\cos(2x - \frac{1}{3}\pi) - \frac{1}{2}x\right]_{\frac{1}{4}\pi}^{\frac{7}{12}\pi}$$

$$= -\frac{1}{2}\cos(\frac{5}{6}\pi) - \frac{7}{24}\pi - (-\frac{1}{2}\cos(\frac{1}{6}\pi) - \frac{1}{8}\pi) = -\frac{1}{2} \cdot -\frac{1}{2}\sqrt{3} - \frac{7}{24}\pi + \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} + \frac{1}{8}\pi$$

$$= \frac{1}{4}\sqrt{3} + \frac{1}{4}\sqrt{3} - \frac{4}{24}\pi = \frac{1}{2}\sqrt{3} - \frac{1}{6}\pi$$

10 Er geldt $\vec{a} = \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix}$, $\vec{b} = \begin{pmatrix} \cos(\beta) \\ \sin(\beta) \end{pmatrix}$ en $\angle(\vec{a}, \vec{b}) = \alpha - \beta$.

$$\cos(\alpha - \beta) = \cos(\angle(\vec{a}, \vec{b})) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{\begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} \cdot \begin{pmatrix} \cos(\beta) \\ \sin(\beta) \end{pmatrix}}{1 \cdot 1} = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

Bladzijde 149

- 11 a $\cos(t - u) = \cos(t)\cos(u) + \sin(t)\sin(u)$
 u vervangen door $-u$ geeft
 $\cos(t + u) = \cos(t)\cos(-u) + \sin(t)\sin(-u)$
 $\cos(t + u) = \cos(t)\cos(u) + \sin(t) \cdot -\sin(u)$
 $\cos(t + u) = \cos(t)\cos(u) - \sin(t)\sin(u)$
- b $\cos(t + u) = \cos(t)\cos(u) - \sin(t)\sin(u)$
 u vervangen door $u - \frac{1}{2}\pi$ geeft
 $\cos(t + u - \frac{1}{2}\pi) = \cos(t)\cos(u - \frac{1}{2}\pi) - \sin(t)\sin(u - \frac{1}{2}\pi)$
 $\sin(t + u) = \cos(t)\sin(u) - \sin(t) \cdot -\cos(u)$
 $\sin(t + u) = \cos(t)\sin(u) + \sin(t)\cos(u)$
 $\sin(t + u) = \sin(t)\cos(u) + \cos(t)\sin(u)$
- c $\sin(t + u) = \sin(t)\cos(u) + \cos(t)\sin(u)$
 u vervangen door $-u$ geeft
 $\sin(t - u) = \sin(t)\cos(-u) + \cos(t)\sin(-u)$
 $\sin(t - u) = \sin(t)\cos(u) + \cos(t) \cdot -\sin(u)$
 $\sin(t - u) = \sin(t)\cos(u) - \cos(t)\sin(u)$

Bladzijde 150

12 a $\sin(t + u) = \sin(t) \cos(u) + \cos(t) \sin(u)$

$t = A$ en $u = A$ geeft

$$\sin(A + A) = \sin(A) \cos(A) + \cos(A) \sin(A)$$

$$\sin(2A) = 2 \sin(A) \cos(A)$$

$$\cos(t + u) = \cos(t) \cos(u) - \sin(t) \sin(u)$$

$t = A$ en $u = A$ geeft

$$\cos(A + A) = \cos(A) \cos(A) - \sin(A) \sin(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

b $\cos(2A) = \cos^2(A) - \sin^2(A)$

$$\cos(2A) = \cos^2(A) - (1 - \cos^2(A))$$

$$\cos(2A) = \cos^2(A) - 1 + \cos^2(A)$$

$$\cos(2A) = 2 \cos^2(A) - 1$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$\cos(2A) = 1 - \sin^2(A) - \sin^2(A)$$

$$\cos(2A) = 1 - 2 \sin^2(A)$$

13 a $\cos(2A) = 2 \cos^2(A) - 1$

$$2 \cos^2(A) = 1 + \cos(2A)$$

$$\cos^2(A) = \frac{1}{2} + \frac{1}{2} \cos(2A)$$

b $\cos(2A) = 1 - 2 \sin^2(A)$

$$2 \sin^2(A) = 1 - \cos(2A)$$

$$\sin^2(A) = \frac{1}{2} - \frac{1}{2} \cos(2A)$$

14 a $\sin(x - \frac{1}{4}\pi) = \sin(x) \cos(\frac{1}{4}\pi) - \cos(x) \sin(\frac{1}{4}\pi) = \sin(x) \cdot \frac{1}{2}\sqrt{2} - \cos(x) \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2}(\sin(x) - \cos(x))$

b $\sin(\frac{1}{12}\pi) = \sin(\frac{1}{3}\pi - \frac{1}{4}\pi) = \sin(\frac{1}{3}\pi) \cos(\frac{1}{4}\pi) - \cos(\frac{1}{3}\pi) \sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{2}$
 $= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$

c $\sin(x + \frac{1}{4}\pi) = \sin(x) \cos(\frac{1}{4}\pi) + \cos(x) \sin(\frac{1}{4}\pi) = \sin(x) \cdot \frac{1}{2}\sqrt{2} + \cos(x) \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2}(\sin(x) + \cos(x))$

d $\sin(\frac{7}{12}\pi) = \sin(\frac{1}{3}\pi + \frac{1}{4}\pi) = \sin(\frac{1}{3}\pi) \cos(\frac{1}{4}\pi) + \cos(\frac{1}{3}\pi) \sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} + \frac{1}{2} \cdot \frac{1}{2}\sqrt{2}$
 $= \frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} + \sqrt{2})$

15 a $\cos(\frac{1}{12}\pi) = \cos(\frac{1}{3}\pi - \frac{1}{4}\pi) = \cos(\frac{1}{3}\pi) \cos(\frac{1}{4}\pi) + \sin(\frac{1}{3}\pi) \sin(\frac{1}{4}\pi) = \frac{1}{2} \cdot \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} = \frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2}$

b $\sin^2(\frac{1}{8}\pi) = \frac{1}{2} - \frac{1}{2}\cos(\frac{1}{4}\pi) = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2} - \frac{1}{4}\sqrt{2}$

$$\sin(\frac{1}{8}\pi) = \sqrt{\frac{1}{2} - \frac{1}{4}\sqrt{2}} \vee \sin(\frac{1}{8}\pi) = -\sqrt{\frac{1}{2} - \frac{1}{4}\sqrt{2}}$$

vold. niet, want $\sin(\frac{1}{8}\pi)$ is positief

16 a $\cos(4x) \cos(x) + \sin(4x) \sin(x) = \cos(4x - x) = \cos(3x)$

b $\sin(\frac{1}{2}x) \cos(x) - \cos(\frac{1}{2}x) \sin(x) = \sin(\frac{1}{2}x - x) = \sin(-\frac{1}{2}x)$

c $\cos(\frac{3}{4}x) \cos(\frac{1}{4}x) - \sin(\frac{3}{4}x) \sin(\frac{1}{4}x) = \cos(\frac{3}{4}x + \frac{1}{4}x) = \cos(x)$

d $\sin(\frac{1}{3}x) \cos(\frac{2}{3}x) + \cos(\frac{1}{3}x) \sin(\frac{2}{3}x) = \sin(\frac{1}{3}x + \frac{2}{3}x) = \sin(x)$

17 $\sin(x)(\sin(4x) - \sin(2x)) + \cos(x)(\cos(4x) + \cos(2x)) =$
 $\sin(x) \sin(4x) - \sin(x) \sin(2x) + \cos(x) \cos(4x) + \cos(x) \cos(2x) =$
 $\cos(4x) \cos(x) + \sin(4x) \sin(x) + \cos(x) \cos(2x) - \sin(x) \sin(2x) =$
 $\cos(4x - x) + \cos(x + 2x) = \cos(3x) + \cos(3x) = 2 \cos(3x)$

18 a $2 \sin(x) \cos(x) = \sin(2x)$

$$\sin(2x) = \sin(x - 1)$$

$$2x = x - 1 + k \cdot 2\pi \vee 2x = \pi - (x - 1) + k \cdot 2\pi$$

$$x = -1 + k \cdot 2\pi \vee 2x = \pi - x + 1 + k \cdot 2\pi$$

$$x = -1 + k \cdot 2\pi \vee 3x = \pi + 1 + k \cdot 2\pi$$

$$x = -1 + k \cdot 2\pi \vee x = \frac{1}{3}\pi + \frac{1}{3} + k \cdot \frac{2}{3}\pi$$

b Gebruik $\cos^2(A) = \frac{1}{2} + \frac{1}{2} \cos(2A)$.

$$\cos^2(2x) = \cos(4x) + \frac{1}{2}$$

$$\frac{1}{2} + \frac{1}{2} \cos(4x) = \cos(4x) + \frac{1}{2}$$

$$\frac{1}{2} \cos(4x) = \cos(4x)$$

$$\cos(4x) = 0$$

$$4x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = \frac{1}{8}\pi + k \cdot \frac{1}{4}\pi$$

c Gebruik $\sin^2(A) = \frac{1}{2} - \frac{1}{2} \cos(2A)$.

$$\sin^2(\frac{1}{2}x) = \cos(x) + 1\frac{1}{4}$$

$$\frac{1}{2} - \frac{1}{2} \cos(x) = \cos(x) + 1\frac{1}{4}$$

$$-1\frac{1}{2} \cos(x) = \frac{3}{4}$$

$$\cos(x) = -\frac{1}{2}$$

$$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$$

d $(\sin(x) + \cos(x))^2 = 1\frac{1}{2}$

$$\sin^2(x) + 2 \sin(x) \cos(x) + \cos^2(x) = 1\frac{1}{2}$$

$$\sin^2(x) + \cos^2(x) + \sin(2x) = 1\frac{1}{2}$$

$$1 + \sin(2x) = 1\frac{1}{2}$$

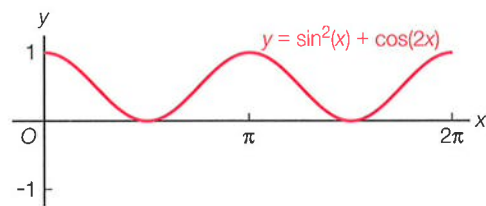
$$\sin(2x) = \frac{1}{2}$$

$$2x = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \pi - \frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot \pi \vee 2x = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot \pi \vee x = \frac{5}{12}\pi + k \cdot \pi$$

19 a



De evenwichtsstand is $\frac{1+0}{2} = \frac{1}{2}$, dus $a = \frac{1}{2}$.

De amplitude is $1 - \frac{1}{2} = \frac{1}{2}$, dus $b = \frac{1}{2}$.

De periode is π , dus $c = \frac{2\pi}{\pi} = 2$.

Een hoogste punt is $(0, 1)$, dus $y = \frac{1}{2} + \frac{1}{2} \cos(2x)$.

b Gebruik $\sin^2(A) = \frac{1}{2} - \frac{1}{2} \cos(2A)$.

$$y = \sin^2(x) + \cos(2x)$$

$$= \frac{1}{2} - \frac{1}{2} \cos(2x) + \cos(2x)$$

$$= \frac{1}{2} + \frac{1}{2} \cos(2x)$$

20 $\sin(3x) = \sin(2x + x)$

$$= \sin(2x) \cos(x) + \cos(2x) \sin(x)$$

$$= 2 \sin(x) \cos(x) \cos(x) + (1 - 2 \sin^2(x)) \cdot \sin(x)$$

$$= 2 \sin(x) \cos^2(x) + \sin(x) - 2 \sin^3(x)$$

$$= 2 \sin(x)(1 - \sin^2(x)) + \sin(x) - 2 \sin^3(x)$$

$$= 2 \sin(x) - 2 \sin^3(x) + \sin(x) - 2 \sin^3(x)$$

$$= 3 \sin(x) - 4 \sin^3(x)$$

Bladzijde 151

21 a Gebruik $\sin^2(A) = \frac{1}{2} - \frac{1}{2} \cos(2A)$.

$$\begin{aligned} y &= 1 - \cos(x) - \sin^2\left(\frac{1}{2}x\right) \\ &= 1 - \cos(x) - \left(\frac{1}{2} - \frac{1}{2}\cos(x)\right) \\ &= 1 - \cos(x) - \frac{1}{2} + \frac{1}{2}\cos(x) \\ &= \frac{1}{2} - \frac{1}{2}\cos(x) \end{aligned}$$

b $\cos(3x) = \cos(2x + x)$

$$\begin{aligned} &= \cos(2x)\cos(x) - \sin(2x)\sin(x) \\ &= (2\cos^2(x) - 1) \cdot \cos(x) - 2\sin(x)\cos(x)\sin(x) \\ &= 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x) \\ &= 2\cos^3(x) - \cos(x) - 2(1 - \cos^2(x)) \cdot \cos(x) \\ &= 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x) \\ &= 4\cos^3(x) - 3\cos(x) \end{aligned}$$

22 a $-\cos(A) = \cos(A + \pi)$

b $\sin(A) = \cos\left(A - \frac{1}{2}\pi\right)$

c $\cos(2A) = 1 - 2\sin^2(A)$ oftewel $\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$

d $\cos(2A) = 2\cos^2(A) - 1$ oftewel $\cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A)$

e $\cos(A) = \sin\left(A + \frac{1}{2}\pi\right)$

12.2 Goniometrische formules bij symmetrie en primitiveren**Bladzijde 153**

23 $x_B = -x_A$ en $y_B = y_A$
 $x_C = -x_A$ en $y_C = -y_A$

Bladzijde 155

24 a In voorbeeld a worden twee functiewaarden apart berekend die achteraf gelijk blijken te zijn.

In voorbeeld b wordt in één keer de som van twee functiewaarden berekend. Bij die berekening vallen verschillende termen tegen elkaar weg.

b $f\left(\frac{1}{4}\pi - p\right) + f\left(\frac{1}{4}\pi + p\right) =$

$$\begin{aligned} &\sin\left(\frac{1}{4}\pi - p\right) - \cos\left(\frac{1}{4}\pi - p\right) + 1\frac{1}{2} + \sin\left(\frac{1}{4}\pi + p\right) - \cos\left(\frac{1}{4}\pi + p\right) + 1\frac{1}{2} = \\ &\cos\left(\frac{1}{4}\pi + p\right) - \sin\left(\frac{1}{4}\pi + p\right) + 1\frac{1}{2} + \sin\left(\frac{1}{4}\pi + p\right) - \cos\left(\frac{1}{4}\pi + p\right) + 1\frac{1}{2} = 3 \end{aligned}$$

Er geldt $f\left(\frac{1}{4}\pi - p\right) + f\left(\frac{1}{4}\pi + p\right) = 2y_A$, dus de grafiek van f is symmetrisch in het punt $A\left(\frac{1}{4}\pi, 1\frac{1}{2}\right)$.

25 a $f(-p) + f(p) = -p \cos(-p) + p \cos(p) = -p \cos(p) + p \cos(p) = 0$
 Er geldt $f(-p) + f(p) = 2 \cdot 0$, dus de grafiek van f is symmetrisch in de oorsprong.

b $g(-p) = -p \sin(-p) = p \sin(p)$
 $g(p) = p \sin(p)$
 Er geldt $g(-p) = g(p)$, dus de grafiek van g is symmetrisch in de y -as.

Bladzijde 156

26 a $f(-p) + f(p) = \cos^2(-p)\sin(-p) + \cos^2(p)\sin(p) = \cos^2(p) \cdot -\sin(p) + \cos^2(p)\sin(p) = 0$
 Er geldt $f(-p) + f(p) = 2 \cdot 0$, dus de grafiek van f is symmetrisch in O .

b $f\left(\frac{1}{2}\pi - p\right) = \cos^2\left(\frac{1}{2}\pi - p\right)\sin\left(\frac{1}{2}\pi - p\right)$

$$\begin{aligned} &= (\cos\left(\frac{1}{2}\pi\right)\cos(p) + \sin\left(\frac{1}{2}\pi\right)\sin(p))^2 (\sin\left(\frac{1}{2}\pi\right)\cos(p) - \cos\left(\frac{1}{2}\pi\right)\sin(p)) \\ &= (0 \cdot \cos(p) + 1 \cdot \sin(p))^2 (1 \cdot \cos(p) - 0 \cdot \sin(p)) = \sin^2(p)\cos(p) \end{aligned}$$

$f\left(\frac{1}{2}\pi + p\right) = \cos^2\left(\frac{1}{2}\pi + p\right)\sin\left(\frac{1}{2}\pi + p\right)$

$$\begin{aligned} &= (\cos\left(\frac{1}{2}\pi\right)\cos(p) - \sin\left(\frac{1}{2}\pi\right)\sin(p))^2 (\sin\left(\frac{1}{2}\pi\right)\cos(p) + \cos\left(\frac{1}{2}\pi\right)\sin(p)) \\ &= (0 \cdot \cos(p) - 1 \cdot \sin(p))^2 (1 \cdot \cos(p) + 0 \cdot \sin(p)) = \sin^2(p)\cos(p) \end{aligned}$$

Er geldt $f\left(\frac{1}{2}\pi - p\right) = f\left(\frac{1}{2}\pi + p\right)$, dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{2}\pi$.

27 a $f(\frac{1}{4}\pi - p) = \cos(\frac{1}{4}\pi - p) + \sin(\frac{1}{4}\pi - p) + 1$
 $= \cos(\frac{1}{4}\pi)\cos(p) + \sin(\frac{1}{4}\pi)\sin(p) + \sin(\frac{1}{4}\pi)\cos(p) - \cos(\frac{1}{4}\pi)\sin(p) + 1$
 $= \frac{1}{2}\sqrt{2}\cos(p) + \frac{1}{2}\sqrt{2}\sin(p) + \frac{1}{2}\sqrt{2}\cos(p) - \frac{1}{2}\sqrt{2}\sin(p) + 1 = \sqrt{2}\cos(p) + 1$
 $f(\frac{1}{4}\pi + p) = \cos(\frac{1}{4}\pi + p) + \sin(\frac{1}{4}\pi + p) + 1$
 $= \cos(\frac{1}{4}\pi)\cos(p) - \sin(\frac{1}{4}\pi)\sin(p) + \sin(\frac{1}{4}\pi)\cos(p) + \cos(\frac{1}{4}\pi)\sin(p) + 1$
 $= \frac{1}{2}\sqrt{2}\cos(p) - \frac{1}{2}\sqrt{2}\sin(p) + \frac{1}{2}\sqrt{2}\cos(p) + \frac{1}{2}\sqrt{2}\sin(p) + 1 = \sqrt{2}\cos(p) + 1$

Er geldt $f(\frac{1}{4}\pi - p) = f(\frac{1}{4}\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{4}\pi$.

b $f(\frac{3}{4}\pi - p) + f(\frac{3}{4}\pi + p) = \cos(\frac{3}{4}\pi - p) + \sin(\frac{3}{4}\pi - p) + 1 + \cos(\frac{3}{4}\pi + p) + \sin(\frac{3}{4}\pi + p) + 1$
 $= \cos(\frac{3}{4}\pi)\cos(p) + \sin(\frac{3}{4}\pi)\sin(p) + \sin(\frac{3}{4}\pi)\cos(p) - \cos(\frac{3}{4}\pi)\sin(p) + 1$
 $+ \cos(\frac{3}{4}\pi)\cos(p) - \sin(\frac{3}{4}\pi)\sin(p) + \sin(\frac{3}{4}\pi)\cos(p) + \cos(\frac{3}{4}\pi)\sin(p) + 1$
 $= 2\cos(\frac{3}{4}\pi)\cos(p) + 2\sin(\frac{3}{4}\pi)\cos(p) + 2 = 2 \cdot \frac{1}{2}\sqrt{2}\cos(p) + 2 \cdot \frac{1}{2}\sqrt{2}\cos(p) + 2 = 2$
 Er geldt $f(\frac{3}{4}\pi - p) + f(\frac{3}{4}\pi + p) = 2y_A$, dus de grafiek van f is symmetrisch in het punt $A(\frac{3}{4}\pi, 1)$.

28 $f(\frac{1}{6}\pi - p) = \sin(\frac{1}{6}\pi - p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi - p)$
 $= \sin(\frac{1}{6}\pi)\cos(p) - \cos(\frac{1}{6}\pi)\sin(p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi)\cos(p) + \sqrt{3} \cdot \sin(\frac{1}{6}\pi)\sin(p)$
 $= \frac{1}{2}\cos(p) - \frac{1}{2}\sqrt{3}\sin(p) + \sqrt{3} \cdot \frac{1}{2}\sqrt{3}\cos(p) + \sqrt{3} \cdot \frac{1}{2}\sin(p) = 2\cos(p)$
 $f(\frac{1}{6}\pi + p) = \sin(\frac{1}{6}\pi + p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi + p)$
 $= \sin(\frac{1}{6}\pi)\cos(p) + \cos(\frac{1}{6}\pi)\sin(p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi)\cos(p) - \sqrt{3} \cdot \sin(\frac{1}{6}\pi)\sin(p)$
 $= \frac{1}{2}\cos(p) + \frac{1}{2}\sqrt{3}\sin(p) + \sqrt{3} \cdot \frac{1}{2}\sqrt{3}\cos(p) - \sqrt{3} \cdot \frac{1}{2}\sin(p) = 2\cos(p)$
 $f(\frac{1}{6}\pi - p) = f(\frac{1}{6}\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{6}\pi$.

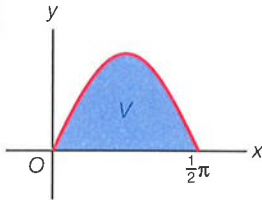
29 a $F(x) = \frac{1}{3}\sin^3(x)$ geeft $F'(x) = \sin^2(x) \cdot \cos(x)$ en dit is niet $f(x)$,
 dus $y = \frac{1}{3}\sin^3(x)$ is geen primitieve van f .
b $\cos(2A) = 1 - 2\sin^2(A)$
 $2\sin^2(A) = 1 - \cos(2A)$
 $\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$
 Dus $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$.
c $f(x) = \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$ geeft $F(x) = \frac{1}{2}x - \frac{1}{4}\sin(2x) + c$

Bladzijde 157

30 a $f(x) = \cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$ geeft $F(x) = \frac{1}{2}x + \frac{1}{4}\sin(2x) + c$
b $g(x) = \sin^2(3x) = \frac{1}{2} - \frac{1}{2}\cos(6x)$ geeft $G(x) = \frac{1}{2}x - \frac{1}{12}\sin(6x) + c$
c $h(x) = \sin(\frac{1}{2}x)\cos(\frac{1}{2}x) = \frac{1}{2}\sin(x)$ geeft $H(x) = -\frac{1}{2}\cos(x) + c$

31 a $\int_0^{\frac{1}{6}\pi} \sin(2x)\cos(2x) dx = \int_0^{\frac{1}{6}\pi} \frac{1}{2}\sin(4x) dx = \left[-\frac{1}{8}\cos(4x)\right]_0^{\frac{1}{6}\pi}$
 $= -\frac{1}{8}\cos(\frac{2}{3}\pi) + \frac{1}{8}\cos(0) = -\frac{1}{8} \cdot -\frac{1}{2} + \frac{1}{8} \cdot 1 = \frac{1}{16} + \frac{1}{8} = \frac{3}{16}$
b $\int_{\frac{1}{3}\pi}^{\pi} (2 - \frac{1}{2}\sin^2(x)) dx = \int_{\frac{1}{3}\pi}^{\pi} (2 - \frac{1}{4} + \frac{1}{4}\cos(2x)) dx = \int_{\frac{1}{3}\pi}^{\pi} (1\frac{3}{4} + \frac{1}{4}\cos(2x)) dx$
 $= \left[1\frac{3}{4}x + \frac{1}{8}\sin(2x)\right]_{\frac{1}{3}\pi}^{\pi} = 1\frac{3}{4}\pi + \frac{1}{8}\sin(2\pi) - (1\frac{3}{4} \cdot \frac{1}{3}\pi + \frac{1}{8}\sin(\frac{2}{3}\pi))$
 $= 1\frac{3}{4}\pi + 0 - (\frac{7}{12}\pi + \frac{1}{8} \cdot \frac{1}{2}\sqrt{3}) = 1\frac{1}{6}\pi - \frac{1}{16}\sqrt{3}$

32



$$\begin{aligned}
 I(L) &= \pi \int_0^{\frac{1}{2}\pi} (f(x))^2 dx = \pi \int_0^{\frac{1}{2}\pi} \sin^2(2x) dx = \pi \int_0^{\frac{1}{2}\pi} \left(\frac{1}{2} - \frac{1}{2}\cos(4x)\right) dx = \pi \left[\frac{1}{2}x - \frac{1}{8}\sin(4x)\right]_0^{\frac{1}{2}\pi} \\
 &= \pi\left(\frac{1}{4}\pi - \frac{1}{8}\sin(2\pi) - \left(0 - \frac{1}{8}\sin(0)\right)\right) = \frac{1}{4}\pi^2
 \end{aligned}$$

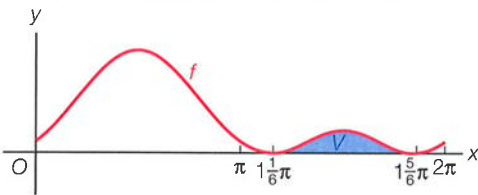
33 $f(x) = 0$ geeft $\sin^2(x) + \sin(x) + \frac{1}{4} = 0$

$$(\sin(x) + \frac{1}{2})^2 = 0$$

$$\sin(x) = -\frac{1}{2}$$

$$x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi$$

x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$



$$\begin{aligned}
 O(V) &= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\sin^2(x) + \sin(x) + \frac{1}{4}) dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\frac{1}{2} - \frac{1}{2}\cos(2x) + \sin(x) + \frac{1}{4}) dx \\
 &= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\frac{3}{4} - \frac{1}{2}\cos(2x) + \sin(x)) dx = \left[\frac{3}{4}x - \frac{1}{4}\sin(2x) - \cos(x)\right]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \\
 &= \frac{11}{8}\pi - \frac{1}{4}\sin(3\frac{2}{3}\pi) - \cos(\frac{5}{6}\pi) - (\frac{7}{8}\pi - \frac{1}{4}\sin(2\frac{1}{3}\pi) - \cos(\frac{1}{6}\pi)) \\
 &= \frac{11}{8}\pi - \frac{1}{4} \cdot -\frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3} - (\frac{7}{8}\pi - \frac{1}{4} \cdot \frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3}) \\
 &= \frac{11}{8}\pi + \frac{1}{8}\sqrt{3} - \frac{1}{2}\sqrt{3} - \frac{7}{8}\pi + \frac{1}{8}\sqrt{3} - \frac{1}{2}\sqrt{3} \\
 &= \frac{1}{2}\pi - \frac{3}{4}\sqrt{3}
 \end{aligned}$$

34 a $f(x) = 2\sin^2(x) + \sin(x) - 1$ geeft $f'(x) = 4\sin(x)\cos(x) + \cos(x)$

$f'(x) = 0$ geeft $4\sin(x)\cos(x) + \cos(x) = 0$

$$\cos(x)(4\sin(x) + 1) = 0$$

$$\cos(x) = 0 \vee \sin(x) = -\frac{1}{4}$$

$\cos(x) = 0$ geeft $x = \frac{1}{2}\pi + k \cdot \pi$, dus $x = \frac{1}{2}\pi \vee x = \frac{3}{2}\pi$.

$$f(\frac{1}{2}\pi) = 2\sin^2(\frac{1}{2}\pi) + \sin(\frac{1}{2}\pi) - 1 = 2 \cdot 1^2 + 1 - 1 = 2$$

$$f(\frac{3}{2}\pi) = 2\sin^2(\frac{3}{2}\pi) + \sin(\frac{3}{2}\pi) - 1 = 2 \cdot (-1)^2 + -1 - 1 = 0$$

$\sin(x) = -\frac{1}{4}$ geeft $f(x) = 2 \cdot (-\frac{1}{4})^2 - \frac{1}{4} - 1 = -\frac{1}{8}$

Dus $B_f = [-\frac{1}{8}, 2]$.

b $f(x) = 0$ geeft $2\sin^2(x) + \sin(x) - 1 = 0$

$$\text{Stel } \sin(x) = u$$

$$2u^2 + u - 1 = 0$$

$$D = 1^2 - 4 \cdot 2 \cdot -1 = 9$$

$$u = \frac{-1 \pm 3}{4} = \frac{1}{2} \vee u = \frac{-1 - 3}{4} = -1$$

$$\sin(x) = \frac{1}{2} \vee \sin(x) = -1$$

$$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = \frac{1}{2}\pi$$

$$O(V) = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (2\sin^2(x) + \sin(x) - 1) dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (1 - \cos(2x) + \sin(x) - 1) dx$$

$$= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (-\cos(2x) + \sin(x)) dx = \left[-\frac{1}{2}\sin(2x) - \cos(x) \right]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}$$

$$= -\frac{1}{2}\sin\left(\frac{5}{3}\pi\right) - \cos\left(\frac{5}{6}\pi\right) - \left(-\frac{1}{2}\sin\left(\frac{1}{3}\pi\right) - \cos\left(\frac{1}{6}\pi\right)\right)$$

$$= -\frac{1}{2} \cdot -\frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3} - \left(-\frac{1}{2} \cdot \frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3}\right)$$

$$= \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} = 1\frac{1}{2}\sqrt{3}$$

35 Domein $[0, 1\frac{1}{2}\pi]$ en $p > 0$, dus $f_p(x) = p \sin(\frac{1}{3}x) \cos(\frac{1}{3}x) \geq 0$.

$$f_p(x) = 0 \text{ geeft } p \sin(\frac{1}{3}x) \cos(\frac{1}{3}x) = 0$$

$$\sin(\frac{1}{3}x) = 0 \vee \cos(\frac{1}{3}x) = 0$$

$$\frac{1}{3}x = k \cdot \pi \vee \frac{1}{3}x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = k \cdot 3\pi \vee x = \frac{1}{2}\pi + k \cdot 3\pi$$

$$x \text{ in } [0, 1\frac{1}{2}\pi] \text{ geeft } x = 0 \vee x = \frac{1}{2}\pi$$

$$O(V_p) = \int_0^{\frac{1}{2}\pi} p \sin(\frac{1}{3}x) \cos(\frac{1}{3}x) dx = \int_0^{\frac{1}{2}\pi} \frac{1}{2}p \sin(\frac{2}{3}x) dx = \left[\frac{1}{2}p \cdot -\frac{3}{2}\cos(\frac{2}{3}x) \right]_0^{\frac{1}{2}\pi} = -\frac{3}{4}p \cos(\pi) + \frac{3}{4}p \cos(0)$$

$$= \frac{3}{4}p + \frac{3}{4}p = 1\frac{1}{2}p$$

$$O(V_p) = 10 \text{ geeft } 1\frac{1}{2}p = 10$$

$$p = 6\frac{2}{3}$$

12.3 Eenparige cirkelbewegingen en harmonische trillingen

Bladzijde 159

36 P bevindt zich voor het eerst weer in $(1, 0)$ als t gelijk is aan 2π .

Dus na 2π seconden.

Bladzijde 160

37 a De formules $\sin^2(A) + \cos^2(A) = 1$ met $A = \frac{1}{2}t$ en met $A = t$ en $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$ met $A = \frac{1}{2}t$ en $B = t$.

b Voor $t = \frac{1}{6}\pi$ is $PQ = \sqrt{2 - 2\cos(\frac{1}{12}\pi)} \approx 0,26$.

c $PQ = 1$ geeft $\sqrt{2 - 2\cos(\frac{1}{2}t)} = 1$

$$2 - 2\cos(\frac{1}{2}t) = 1$$

$$-2\cos(\frac{1}{2}t) = -1$$

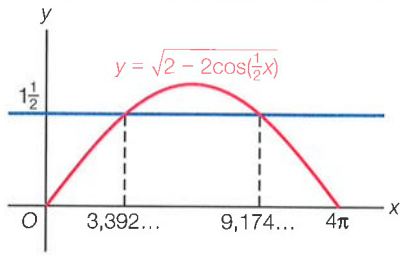
$$\cos(\frac{1}{2}t) = \frac{1}{2}$$

$$\frac{1}{2}t = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{1}{2}t = -\frac{1}{3}\pi + k \cdot 2\pi$$

$$t = \frac{2}{3}\pi + k \cdot 4\pi \vee t = -\frac{2}{3}\pi + k \cdot 4\pi$$

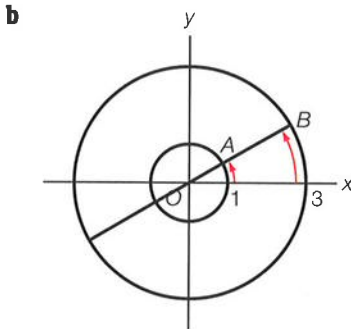
$$t \text{ in } [0, 4\pi] \text{ geeft } t = \frac{2}{3}\pi \vee t = 3\frac{1}{3}\pi$$

- d Voer in $y_1 = \sqrt{2 - 2\cos(\frac{1}{2}x)}$ en $y_2 = 1\frac{1}{2}$.
De optie snijpunt geeft $x = 3,392\dots$ en $x = 9,174\dots$



$PQ > 1\frac{1}{2}$ geeft $3,39 < t < 9,17$

- 38 a $t = \frac{1}{4}\pi$ geeft $A(\cos(\frac{1}{2}\pi), \sin(\frac{1}{2}\pi)) = A(0, 1)$ en $B(3\cos(\frac{1}{4}\pi), 3\sin(\frac{1}{4}\pi)) = B(1\frac{1}{2}\sqrt{2}, 1\frac{1}{2}\sqrt{2})$
 $AB = \sqrt{(1\frac{1}{2}\sqrt{2} - 0)^2 + (1\frac{1}{2}\sqrt{2} - 1)^2} = \sqrt{4\frac{1}{2} + 4\frac{1}{2} - 3\sqrt{2} + 1} = \sqrt{10 - 3\sqrt{2}}$



De kleinste afstand tussen A en B is 2 en die ontstaat als de draaiingshoek van A gelijk is aan de draaiingshoek van B .

Dit geeft $2t = t + k \cdot 2\pi$

$$t = k \cdot 2\pi$$

$$t = 0 \vee t = 2\pi$$

De grootste afstand tussen A en B is 4 en die ontstaat als de draaiingshoek van A gelijk is aan de draaiingshoek van B plus π .

Dit geeft $2t = t + \pi + k \cdot 2\pi$

$$t = \pi + k \cdot 2\pi$$

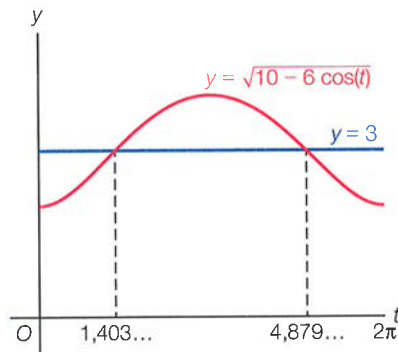
$$t = \pi$$

- c $AB^2 = (3\cos(t) - \cos(2t))^2 + (3\sin(t) - \sin(2t))^2$
 $= 9\cos^2(t) - 6\cos(t)\cos(2t) + \cos^2(2t) + 9\sin^2(t) - 6\sin(t)\sin(2t) + \sin^2(2t)$
 $= 9 + 1 - 6(\cos(2t)\cos(t) + \sin(2t)\sin(t)) = 10 - 6\cos(2t - t)$
 $= 10 - 6\cos(t)$

$$\text{Dus } AB = \sqrt{10 - 6\cos(t)}.$$

- d $\sqrt{10 - 6\cos(t)} = \sqrt{7}$
 $10 - 6\cos(t) = 7$
 $6\cos(t) = 3$
 $\cos(t) = \frac{1}{2}$
 $t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$
 $0 \leq t \leq 2\pi$ geeft $t = \frac{1}{3}\pi \vee t = \frac{5}{3}\pi$

- e Voer in $y_1 = \sqrt{10 - 6 \cos(x)}$ en $y_2 = 3$.
De optie snijpunt geeft $x = 1,403...$ en $x = 4,879...$



$AB < 3$ geeft $0 \leq t < 1,40 \vee 4,88 < t \leq 2\pi$

Bladzijde 161

- 39 a $y = \frac{1}{2}\sqrt{2}$ geeft op de eenheidscirkel punten met richtingshoek $\frac{1}{4}\pi$ en met richtingshoek $\frac{3}{4}\pi$.
Tussen deze twee punten ligt precies een kwart van de eenheidscirkel. De lengte van de baan is dus $\frac{1}{4} \cdot 2\pi \cdot 1 = \frac{1}{2}\pi$.

- b Voor A geldt $\sin(2t) = \frac{2}{3}\cos^2(2t)$
 $\sin(2t) = \frac{2}{3}(1 - \sin^2(2t))$
 $\sin(2t) = \frac{2}{3} - \frac{2}{3}\sin^2(2t)$
 $1\frac{1}{2}\sin(2t) = 1 - \sin^2(2t)$
 $\sin^2(2t) + 1\frac{1}{2}\sin(2t) - 1 = 0$
 $(\sin(2t) - \frac{1}{2})(\sin(2t) + 2) = 0$
 $\sin(2t) = \frac{1}{2} \vee \sin(2t) = -2$
 $2t = \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \frac{5}{6}\pi + k \cdot 2\pi$
 $t = \frac{1}{12}\pi + k \cdot \pi \vee t = \frac{5}{12}\pi + k \cdot \pi$

$P(\cos(\frac{1}{6}\pi), \sin(\frac{1}{6}\pi))$ oftewel $P(\frac{1}{2}\sqrt{3}, \frac{1}{2})$
 $Q(\cos(\frac{5}{6}\pi), \sin(\frac{5}{6}\pi))$ oftewel $Q(-\frac{1}{2}\sqrt{3}, \frac{1}{2})$

- c Voor B geldt $3 \sin(t) = \frac{2}{3} \cdot (3 \cos(t))^2$
 $3 \sin(t) = 6 \cos^2(t)$
 $\sin(t) = 2 \cos^2(t)$

Voer in $y_1 = \sin(x)$ en $y_2 = 2 \cos^2(x)$.

De optie snijpunt geeft $x = 0,895...$ en $x = 2,245...$

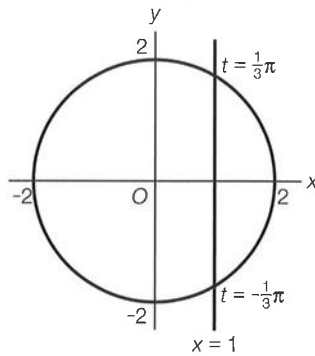
Dus $t = 0,895...$ en $t = 2,245...$

$R(3 \cos(0,895...), 3 \sin(0,895...))$ oftewel $R(1,874; 2,342)$

$S(3 \cos(2,245...), 3 \sin(2,245...))$ oftewel $S(-1,874; 2,342)$

- 40 a Substitutie van $x = 2 \cos(t)$ en $y = 2 \sin(t)$ in $y = x + 1$ geeft $2 \sin(t) = 2 \cos(t) + 1$.
Voer in $y_1 = 2 \sin(x)$ en $y_2 = 2 \cos(x) + 1$.
De optie snijpunt geeft $x \approx 1,15$ en $y \approx 1,82$ en $x \approx 3,57$ en $y \approx -0,82$.
Voor $t \approx 1,15$ is $y \approx 1,82$ en $x = y - 1 \approx 0,82$.
Voor $t \approx 3,57$ is $y \approx -0,82$ en $x = y - 1 \approx -1,82$.
Dit geeft de snijpunten $(0,82; 1,82)$ en $(-1,82; -0,82)$.

- b** $x = 1$ geeft $2 \cos(t) = 1$
 $\cos(t) = \frac{1}{2}$
 $t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$



De baan ligt rechts van de lijn $x = 1$ indien $2 \cos(t) > 1$, dus als $\cos(t) > \frac{1}{2}$ en dit geeft $-\frac{1}{3}\pi < t < \frac{1}{3}\pi$.

De baan is de cirkel met middelpunt $(0, 0)$ en straal 2.

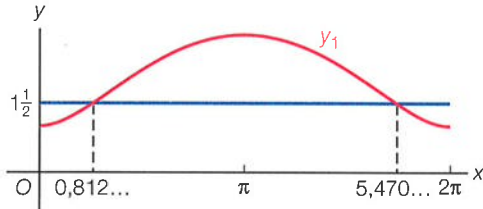
Dus ligt $\frac{\frac{2}{3}\pi}{2\pi} = \frac{1}{3}$ deel van de cirkel rechts van de lijn $x = 1$.

De lengte van dit deel is $\frac{1}{3} \cdot 2\pi \cdot 2 = 1\frac{1}{3}\pi$.

- c** $P(2 \cos(t), 2 \sin(t))$ en $Q(\cos(2t), \sin(2t))$ geeft
 $PQ^2 = (2 \cos(t) - \cos(2t))^2 + (2 \sin(t) - \sin(2t))^2$
 $= 4 \cos^2(t) - 4 \cos(t) \cos(2t) + \cos^2(2t) + 4 \sin^2(t) - 4 \sin(t) \sin(2t) + \sin^2(2t)$
 $= 4 \cos^2(t) + 4 \sin^2(t) + \cos^2(2t) + \sin^2(2t) - 4 \cos(t) \cos(2t) - 4 \sin(t) \sin(2t)$
 $= 4 + 1 - 4(\cos(2t) \cos(t) + \sin(2t) \sin(t))$
 $= 5 - 4 \cos(2t - t)$
 $= 5 - 4 \cos(t)$

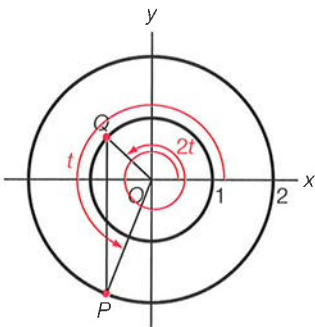
Dus $PQ = \sqrt{5 - 4 \cos(t)}$.

- d** Voer in $y_1 = \sqrt{5 - 4 \cos(x)}$ en $y_2 = 1,5$.
 De optie snijpunt geeft $x = 0,812...$ en $x = 5,470...$



Dus gedurende $5,470... - 0,812... \approx 4,66$ seconde per rondgang.

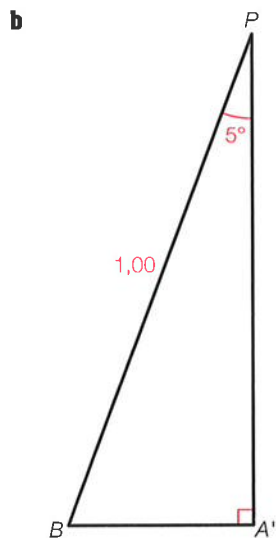
- e** $x_P = x_Q$ geeft $2 \cos(t) = \cos(2t)$
 Voer in $y_1 = 2 \cos(x)$ en $y_2 = \cos(2x)$.
 De optie snijpunt geeft $x = 1,9455...$ en $x = 4,3376...$
 Voor de getekende situatie geldt $t \approx 1,946$.
- f** Voor de andere situatie geldt $t \approx 4,338$.



41
$$\left. \begin{aligned} y_{P'} &= y_P \\ y_P &= 3 \sin\left(\frac{2}{5}\pi t\right) \end{aligned} \right\} y_{P'} = 3 \sin\left(\frac{2}{5}\pi t\right)$$

Bladzijde 163

- 42 a** De frequentie is $\frac{50\pi}{2\pi} = 25$ Hz.
- b** Per trilling is de afgelegde afstand $4 \cdot 5 = 20$ cm.
Er zijn 25 trillingen per seconde, dus per kwartier zijn er $15 \cdot 60 \cdot 25 = 22\,500$ trillingen.
De afgelegde afstand in één kwartier is $22\,500 \cdot 20 = 450\,000$ cm = 4,5 km.
- 43** amplitude = 10 geeft $b = 10$
frequentie = 3 Hz geeft $c = 2\pi \cdot 3 = 6\pi$
Dus $u = 10 \sin(6\pi t)$.
- 44 a** De omtrek van de cirkel met middelpunt P en straal 1,00 m is $2\pi \cdot 1 = 2\pi$ m.
De lengte van boog BC is $\frac{10}{360} \cdot 2\pi \approx 0,1745$ m.



$$\sin(5^\circ) = \frac{A'B}{1}, \text{ dus } A'B = \sin(5^\circ).$$

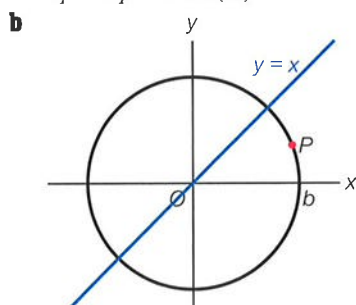
$$BC = 2A'B = 2 \sin(5^\circ) \approx 0,1743 \text{ m}$$

c $b = A'B = \sin(5^\circ) \approx 0,09$

$$c = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{l}{g}}} = \sqrt{\frac{g}{l}} = \sqrt{\frac{9,81}{1}} \approx 3,13$$

d $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{9,81}} \approx 2,01$ seconde, dus de klok geeft ongeveer 1 tik per seconde.

- 45 a** Door de figuur een kwartslag te draaien, zie je dat de projectie van P op de x -as op hetzelfde neerkomt als de projectie van een eenparige cirkelbeweging op de y -as.
Dus P'' voert een harmonische trilling uit.
 $x_{P''} = x_P = b \cos(ct)$



Door de figuur een achtste slag te draaien, zie je dat de projectie van P op de lijn $y = x$ op hetzelfde neerkomt als de projectie van een eenparige cirkelbeweging op de y -as.
Dus de projectie van P op de lijn $y = x$ voert een harmonische trilling uit.

12.4 Bewegingsvergelijkingen met goniometrische formules

Bladzijde 165

- 46 a** Snijden met de x -as, dus $y = 0$.

$$y = 0 \text{ geeft } \sin(4t) = 0$$

$$4t = k \cdot \pi$$

$$t = k \cdot \frac{1}{4}\pi$$

t in $[0, 2\pi]$ geeft $t = 0, t = \frac{1}{4}\pi, t = \frac{1}{2}\pi, t = \frac{3}{4}\pi, t = \pi, t = 1\frac{1}{4}\pi, t = 1\frac{1}{2}\pi, t = 1\frac{3}{4}\pi$ en $t = 2\pi$

Het punt P gaat door de oorsprong voor $t = 0, t = \pi$ en $t = 2\pi$

en snijdt de x -as in $(-1, 0)$ voor $t = 1\frac{1}{2}\pi$ en in $(1, 0)$ voor $t = \frac{1}{2}\pi$.

$$t = \frac{1}{4}\pi \text{ en } t = \frac{3}{4}\pi \text{ geven } x = \sin(\frac{1}{4}\pi) = \sin(\frac{3}{4}\pi) = \frac{1}{2}\sqrt{2}, \text{ dus } x_B = \frac{1}{2}\sqrt{2}.$$

$$t = 1\frac{1}{4}\pi \text{ en } t = 1\frac{3}{4}\pi \text{ geven } x = \sin(1\frac{1}{4}\pi) = \sin(1\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}, \text{ dus } x_A = -\frac{1}{2}\sqrt{2}.$$

- b** C is een top, dus $y_C = 1$.

$$y = 1 \text{ geeft } \sin(4t) = 1$$

$$4t = \frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$$

$$t = \frac{1}{8}\pi \text{ geeft } x = \sin(\frac{1}{8}\pi) = 0,3826\dots, \text{ dus } x_C \approx 0,383.$$

Bladzijde 166

- 47 a** $x = \sin(2t)$ en $y = \sin(t)$ substitueren in $y = -x + 1$ geeft $\sin(t) = -\sin(2t) + 1$.

Voer in $y_1 = \sin(x)$ en $y_2 = -\sin(2x) + 1$.

De optie snijpunt geeft $x = 0,3552\dots$ en $x = 1,5707\dots$

$t = 1,5707\dots$ geeft $\sin(2t) = 0$ en $\sin(t) = 1$, dus het punt $(0, 1)$.

$t = 0,3552\dots$ geeft $\sin(2t) = 0,6521\dots$ en $\sin(t) = 0,3478\dots$, dus het punt $S(0,652; 0,348)$.

- b** De baansnelheid in O is $v(0) = \sqrt{(2 \cos(0))^2 + (\cos(0))^2} = \sqrt{(2 \cdot 1)^2 + 1^2} = \sqrt{5}$.

c
$$\vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2 \cos(0) \\ \cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\tan(\varphi) = \frac{1}{2}, \text{ dus } \varphi \approx 26,6^\circ.$$

Bladzijde 167

- 48 a** $y = \frac{1}{2}$ geeft $\sin(t) = \frac{1}{2}$

$$t = \frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{5}{6}\pi + k \cdot 2\pi$$

t in $[0, 2\pi]$ geeft $t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi$

$$x(\frac{1}{6}\pi) = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}, \text{ dus } x_D = \frac{1}{2}\sqrt{3}.$$

$$x(\frac{5}{6}\pi) = \sin(1\frac{2}{3}\pi) = -\frac{1}{2}\sqrt{3}, \text{ dus } x_C = -\frac{1}{2}\sqrt{3}.$$

$$CD = x_D - x_C = \frac{1}{2}\sqrt{3} - (-\frac{1}{2}\sqrt{3}) = \sqrt{3}$$

- b** De baansnelheid in D is

$$v(\frac{1}{6}\pi) = \sqrt{(2 \cos(\frac{1}{3}\pi))^2 + (\cos(\frac{1}{6}\pi))^2} = \sqrt{(2 \cdot \frac{1}{2})^2 + (\frac{1}{2}\sqrt{3})^2} = \sqrt{1 + \frac{3}{4}} = \sqrt{\frac{7}{4}} = \frac{1}{2}\sqrt{7}.$$

c
$$\vec{v}(\frac{1}{6}\pi) = \begin{pmatrix} x'(\frac{1}{6}\pi) \\ y'(\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2}\sqrt{3} \end{pmatrix} \text{ en } \vec{r}_{y=\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\cos(\alpha) = \frac{\left| \begin{pmatrix} 1 \\ \frac{1}{2}\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ \frac{1}{2}\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|} = \frac{|1 \cdot 1 + \frac{1}{2}\sqrt{3} \cdot 0|}{\sqrt{1^2 + (\frac{1}{2}\sqrt{3})^2} \cdot 1} = \frac{1}{\sqrt{1\frac{3}{4}}}$$

Dus $\alpha \approx 40,9^\circ$.

d $x(t) = 0$ geeft $\sin(2t) = 0$

$$2t = k \cdot \pi$$

$$t = k \cdot \frac{1}{2} \pi$$

$y(t) = 0$ geeft $\sin(t) = 0$

$$t = k \cdot \pi$$

Dus de baan gaat in $[0, 2\pi]$ door de oorsprong voor $t = 0$, $t = \pi$ en $t = 2\pi$.

$$\vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2 \cos(0) \\ \cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ en } \vec{v}(\pi) = \begin{pmatrix} 2 \cos(2\pi) \\ \cos(\pi) \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\cos(\beta) = \frac{\left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right|} = \frac{|2 \cdot 2 + 1 \cdot (-1)|}{\sqrt{2^2 + 1^2} \cdot \sqrt{2^2 + (-1)^2}} = \frac{|4 - 1|}{\sqrt{5} \cdot \sqrt{5}} = \frac{3}{5}$$

Dus $\beta \approx 53,1^\circ$.

49 a $y = 1$ geeft $\sin(3t) = 1$

$$3t = \frac{1}{2} \pi + k \cdot 2\pi$$

$$t = \frac{1}{6} \pi + k \cdot \frac{2}{3} \pi$$

$t = \frac{5}{6} \pi$ geeft $A(\sin(\frac{1}{3} \pi), \sin(2 \cdot \frac{1}{3} \pi)) = A(-\frac{1}{2} \sqrt{3}, 1)$

$x = -1$ geeft $\sin(2t) = -1$

$$2t = \frac{1}{2} \pi + k \cdot 2\pi$$

$$t = \frac{3}{4} \pi + k \cdot \pi$$

$t = \frac{3}{4} \pi$ geeft $B(\sin(\frac{1}{2} \pi), \sin(2 \cdot \frac{1}{2} \pi)) = B(-1, \frac{1}{2} \sqrt{2})$

b $x(t) = \sin(2t)$ en $y(t) = \sin(3t)$ substitueren in $y = x$ geeft

$$\sin(3t) = \sin(2t)$$

$$3t = 2t + k \cdot 2\pi \vee 3t = \pi - 2t + k \cdot 2\pi$$

$$t = k \cdot 2\pi \vee 5t = \pi + k \cdot 2\pi$$

$$t = k \cdot 2\pi \vee t = \frac{1}{5} \pi + k \cdot \frac{2}{5} \pi$$

t in $[0, 2\pi]$ geeft $t = 0 \vee t = \frac{1}{5} \pi \vee t = \frac{3}{5} \pi \vee t = \pi \vee t = \frac{7}{5} \pi \vee t = \frac{9}{5} \pi \vee t = 2\pi$

Bij de oorsprong horen $t = 0$, $t = \pi$ en $t = 2\pi$.

Bij de andere punten horen $t = \frac{1}{5} \pi$, $t = \frac{3}{5} \pi$, $t = \frac{7}{5} \pi$ en $t = \frac{9}{5} \pi$.

c $x(t) = \frac{1}{2}$ geeft $\sin(2t) = \frac{1}{2}$

$$2t = \frac{1}{6} \pi + k \cdot 2\pi \vee 2t = \frac{5}{6} \pi + k \cdot 2\pi$$

$$t = \frac{1}{12} \pi + k \cdot \pi \vee t = \frac{5}{12} \pi + k \cdot \pi$$

t in $[0, 2\pi]$ geeft $t = \frac{1}{12} \pi \vee t = \frac{5}{12} \pi \vee t = \frac{13}{12} \pi \vee t = \frac{17}{12} \pi$

$y(t) = \frac{1}{2} \sqrt{2}$ geeft $\sin(3t) = \frac{1}{2} \sqrt{2}$

$$3t = \frac{1}{4} \pi + k \cdot 2\pi \vee 3t = \frac{3}{4} \pi + k \cdot 2\pi$$

$$t = \frac{1}{12} \pi + k \cdot \frac{2}{3} \pi \vee t = \frac{1}{4} \pi + k \cdot \frac{2}{3} \pi$$

t in $[0, 2\pi]$ geeft $t = \frac{1}{12} \pi \vee t = \frac{3}{4} \pi \vee t = \frac{5}{12} \pi \vee t = \frac{1}{4} \pi \vee t = \frac{11}{12} \pi \vee t = \frac{7}{12} \pi$

Dus voor $t = \frac{1}{12} \pi$ en $t = \frac{5}{12} \pi$ passeert P het punt $(\frac{1}{2}, \frac{1}{2} \sqrt{2})$,

dus de baan snijdt zichzelf in $C(\frac{1}{2}, \frac{1}{2} \sqrt{2})$.

d $x(t) = \sin(2t)$ geeft $x'(t) = 2 \cos(2t)$

$y(t) = \sin(3t)$ geeft $y'(t) = 3 \cos(3t)$

P is voor het eerst in C op $t = \frac{1}{12} \pi$.

De baansnelheid in C is dan

$$v(\frac{1}{12} \pi) = \sqrt{(2 \cos(\frac{1}{6} \pi))^2 + (3 \cos(\frac{1}{4} \pi))^2} = \sqrt{(2 \cdot \frac{1}{2} \sqrt{3})^2 + (3 \cdot \frac{1}{2} \sqrt{2})^2} = \sqrt{3 + 4 \frac{1}{2}} = \sqrt{7 \frac{1}{2}} = \frac{1}{2} \sqrt{30}.$$

e $\vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2 \cos(0) \\ 3 \cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ en $\vec{v}(\pi) = \begin{pmatrix} x'(\pi) \\ y'(\pi) \end{pmatrix} = \begin{pmatrix} 2 \cos(2\pi) \\ 3 \cos(3\pi) \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right|} = \frac{|4 - 9|}{\sqrt{2^2 + 3^2} \cdot \sqrt{2^2 + (-3)^2}} = \frac{|-5|}{\sqrt{13} \cdot \sqrt{13}} = \frac{5}{13}$$

Dus $\varphi \approx 67^\circ$.

50 a $x = 1$ geeft $\sin(t - \frac{1}{6}\pi) = 1$

$$t - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{2}{3}\pi + k \cdot 2\pi$$

t in $[0, 2\pi]$ geeft $t = \frac{2}{3}\pi$

$x(\frac{2}{3}\pi) = \sin(\frac{1}{2}\pi) = 1$ en $y(\frac{2}{3}\pi) = \sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$, dus $A(1, -\frac{1}{2}\sqrt{3})$.

b $x = \frac{1}{2}$ geeft $\sin(t - \frac{1}{6}\pi) = \frac{1}{2}$

$$t - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee t - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi$$

$$t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = \pi + k \cdot 2\pi$$

t in $[0, 2\pi]$ geeft $t = \frac{1}{3}\pi \vee t = \pi$

$y(\frac{1}{3}\pi) = \sin(\frac{2}{3}\pi) = \frac{1}{2}\sqrt{3}$ geeft $B(\frac{1}{2}, \frac{1}{2}\sqrt{3})$

$y(\pi) = \sin(2\pi) = 0$ geeft $C(\frac{1}{2}, 0)$

Dus $BC = \frac{1}{2}\sqrt{3}$.

c $x(t) = \sin(t - \frac{1}{6}\pi)$ en $y(t) = \sin(2t)$ substitueren in $y = x$ geeft

$$\sin(2t) = \sin(t - \frac{1}{6}\pi)$$

$$2t = t - \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \pi - (t - \frac{1}{6}\pi) + k \cdot 2\pi$$

$$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \pi - t + \frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee 3t = 1\frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$$

t in $[0, 2\pi]$ geeft $t = 1\frac{5}{6}\pi \vee t = \frac{7}{18}\pi \vee t = 1\frac{1}{18}\pi \vee t = 1\frac{13}{18}\pi$

d $x(t) = 0$ geeft $\sin(t - \frac{1}{6}\pi) = 0$

$$t - \frac{1}{6}\pi = k \cdot \pi$$

$$t = \frac{1}{6}\pi + k \cdot \pi$$

Dus de baan snijdt zichzelf voor $t = \frac{1}{6}\pi$ en voor $t = 1\frac{1}{6}\pi$ in $(0, \frac{1}{2}\sqrt{3})$.

$x(t) = \sin(t - \frac{1}{6}\pi)$ geeft $x'(t) = \cos(t - \frac{1}{6}\pi)$

$y(t) = \sin(2t)$ geeft $y'(t) = 2\cos(2t)$

$$\vec{v}(\frac{1}{6}\pi) = \begin{pmatrix} x'(\frac{1}{6}\pi) \\ y'(\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} \cos(0) \\ 2\cos(\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ en } \vec{v}(1\frac{1}{6}\pi) = \begin{pmatrix} x'(1\frac{1}{6}\pi) \\ y'(1\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} \cos(\pi) \\ 2\cos(2\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 0, \text{ dus de hoek waaronder de baan zichzelf snijdt is } 90^\circ.$$

e De baansnelheid op $t = \frac{1}{3}\pi$ is

$$v(\frac{1}{3}\pi) = \sqrt{(\cos(\frac{1}{6}\pi))^2 + (2\cos(\frac{2}{3}\pi))^2} = \sqrt{(\frac{1}{2}\sqrt{3})^2 + (2 \cdot -\frac{1}{2})^2} = \sqrt{\frac{3}{4} + 1} = \sqrt{\frac{7}{4}} = \frac{1}{2}\sqrt{7}.$$

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51 a $x = 0$ geeft $\sin(2t) = 0$

$$2t = k \cdot \pi$$

$$t = k \cdot \frac{1}{2}\pi$$

t in $[0, 2\pi]$ geeft $t = 0 \vee t = \frac{1}{2}\pi \vee t = \pi \vee t = 1\frac{1}{2}\pi \vee t = 2\pi$

$y(0) = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$

$y(\frac{1}{2}\pi) = \sin(\frac{5}{6}\pi) = \frac{1}{2}$

$y(\pi) = \sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$

$y(1\frac{1}{2}\pi) = \sin(1\frac{5}{6}\pi) = -\frac{1}{2}$

$y(2\pi) = \sin(2\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$

Dus $A(0, \frac{1}{2}\sqrt{3})$, $B(0, \frac{1}{2})$, $C(0, -\frac{1}{2}\sqrt{3})$ en $D(0, -\frac{1}{2}\sqrt{3})$.

b $x = -\frac{1}{2}$ geeft $\sin(2t) = -\frac{1}{2}$

$$2t = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \pi + \frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{1}{12}\pi + k \cdot \pi \vee t = \frac{7}{12}\pi + k \cdot \pi$$

t in $[0, 2\pi]$ geeft $t = \frac{7}{12}\pi \vee t = \frac{11}{12}\pi \vee t = 1\frac{7}{12}\pi \vee t = 1\frac{11}{12}\pi$

$$y(\frac{7}{12}\pi) = \sin(\frac{11}{12}\pi) \approx 0,26$$

$$y(\frac{11}{12}\pi) = \sin(1\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$$

$$y(1\frac{7}{12}\pi) = \sin(1\frac{11}{12}\pi) \approx -0,26$$

$$y(1\frac{11}{12}\pi) = \sin(2\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2}$$

Dus $E(-\frac{1}{2}, \frac{1}{2}\sqrt{2})$ en $H(-\frac{1}{2}, -\frac{1}{2}\sqrt{2})$ en dit geeft $EH = \frac{1}{2}\sqrt{2} - (-\frac{1}{2}\sqrt{2}) = \sqrt{2}$.

c $x(t) = \sin(2t)$ geeft $x'(t) = 2\cos(2t)$

$$y(t) = \sin(t + \frac{1}{3}\pi) \text{ geeft } y'(t) = \cos(t + \frac{1}{3}\pi)$$

$$\vec{v}(\frac{1}{2}\pi) = \begin{pmatrix} x'(\frac{1}{2}\pi) \\ y'(\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} 2\cos(\pi) \\ \cos(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \text{ en } \vec{r}_{y-as} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|}{\left| \begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|} = \frac{\left| -\frac{1}{2}\sqrt{3} \right|}{\sqrt{(-2)^2 + (-\frac{1}{2}\sqrt{3})^2} \cdot 1} = \frac{\frac{1}{2}\sqrt{3}}{\sqrt{4 + \frac{3}{4}}} = \frac{\frac{1}{2}\sqrt{3}}{\sqrt{4\frac{3}{4}}}$$

Dus $\varphi \approx 67^\circ$.

d De baansnelheid in E is

$$v(1\frac{11}{12}\pi) = \sqrt{(2\cos(3\frac{5}{6}\pi))^2 + (\cos(2\frac{1}{4}\pi))^2} = \sqrt{4 \cdot (\frac{1}{2}\sqrt{3})^2 + (\frac{1}{2}\sqrt{2})^2} = \sqrt{3 + \frac{1}{2}} = \frac{1}{2}\sqrt{14}.$$

52 a $x(a) = \sin(2a)$ en $y(a) = \sin(a + \frac{1}{3}\pi)$, dus $T(\sin(2a), \sin(a + \frac{1}{3}\pi))$.

$x(a + \pi) = \sin(2(a + \pi)) = \sin(2a + 2\pi) = \sin(2a)$ en $y(a + \pi) = \sin(a + \pi + \frac{1}{3}\pi) = \sin(a + 1\frac{1}{3}\pi)$, dus $U(\sin(2a), \sin(a + 1\frac{1}{3}\pi))$.

$$TU = |y_T - y_U| = \left| \sin(a + \frac{1}{3}\pi) - \sin(a + 1\frac{1}{3}\pi) \right| = \left| \sin(a + \frac{1}{3}\pi) - (-\sin(a + \frac{1}{3}\pi)) \right| = \left| 2\sin(a + \frac{1}{3}\pi) \right|$$

b $|2\sin(a + \frac{1}{3}\pi)| = 1$

$$2\sin(a + \frac{1}{3}\pi) = 1 \vee 2\sin(a + \frac{1}{3}\pi) = -1$$

$$\sin(a + \frac{1}{3}\pi) = \frac{1}{2} \vee \sin(a + \frac{1}{3}\pi) = -\frac{1}{2}$$

$$a + \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi$$

$$a = -\frac{1}{6}\pi + k \cdot 2\pi \vee a = \frac{1}{2}\pi + k \cdot 2\pi \vee a = -\frac{1}{2}\pi + k \cdot 2\pi \vee a = \frac{5}{6}\pi + k \cdot 2\pi$$

$0 \leq a \leq 2\pi$ en $0 \leq a + \pi \leq 2\pi$ oftewel $0 \leq a \leq \pi$ geeft $a = \frac{1}{2}\pi \vee a = \frac{5}{6}\pi$.

53 a

t	$\frac{1}{4}\pi$	$\frac{3}{4}\pi$	$1\frac{1}{4}\pi$
x	1	0	-1
y	1	-1	1

b $y = px^2 + q \left\{ \begin{array}{l} p \cdot 0^2 + q = -1 \\ q = -1 \end{array} \right.$

$y = px^2 - 1 \left\{ \begin{array}{l} p \cdot 1^2 - 1 = 1 \\ p = 2 \end{array} \right.$

Dus $p = 2$ en $q = -1$.

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- 54**
- Substitutie van
- $x = \sin(t - \frac{1}{4}\pi)$
- en
- $y = \sin(2t)$
- in
- $y = -2x^2 + 1$
- geeft

$$\sin(2t) = -2 \sin^2(t - \frac{1}{4}\pi) + 1$$

$$\sin(2t) = 1 - 2 \sin^2(t - \frac{1}{4}\pi)$$

$$\sin(2t) = \cos(2(t - \frac{1}{4}\pi))$$

$$\sin(2t) = \cos(2t - \frac{1}{2}\pi)$$

$$\sin(2t) = \sin(2t - \frac{1}{2}\pi + \frac{1}{2}\pi)$$

$$\sin(2t) = \sin(2t)$$

Dit klopt voor elke t .

$$\left. \begin{array}{l} x = \sin(t - \frac{1}{4}\pi) \\ -1 \leq \sin(t - \frac{1}{4}\pi) \leq 1 \end{array} \right\} \text{Bij de parametervoorstelling hoort de formule } y = -2x^2 + 1 \text{ met } -1 \leq x \leq 1.$$

- 55 a**
- $-2 \leq 2 \sin(t) \leq 2$
- , dus
- $-2 \leq x \leq 2$
- .

$$-1 \leq \sin(2t - \frac{1}{2}\pi) \leq 1, \text{ dus } -1 \leq y \leq 1.$$

Dus de keerpunten zijn $(-2, 1)$ en $(2, 1)$.

- b**
- Stel
- $K: y = ax^2 + b$
- .

$$x = 0 \text{ geeft } t = 0 \text{ en } y(0) = \sin(-\frac{1}{2}\pi) = -1, \text{ dus door } (0, -1).$$

$$\left. \begin{array}{l} y = ax^2 + b \\ \text{door } (0, -1) \end{array} \right\} \begin{array}{l} a \cdot 0^2 + b = -1 \\ b = -1 \end{array}$$

$$\left. \begin{array}{l} y = ax^2 - 1 \\ \text{door } (2, 1) \end{array} \right\} \begin{array}{l} a \cdot 2^2 - 1 = 1 \\ 4a = 2 \\ a = \frac{1}{2} \end{array}$$

Vermoedelijk hoort bij K de formule $y = \frac{1}{2}x^2 - 1$ met $-2 \leq x \leq 2$.

Substitutie van $x = 2 \sin(t)$ en $y = \sin(2t - \frac{1}{2}\pi)$ in $y = \frac{1}{2}x^2 - 1$ geeft

$$\sin(2t - \frac{1}{2}\pi) = \frac{1}{2}(2 \sin(t))^2 - 1$$

$$\cos(2t - \frac{1}{2}\pi - \frac{1}{2}\pi) = \frac{1}{2} \cdot 4 \sin^2(t) - 1$$

$$\cos(2t - \pi) = 2 \sin^2(t) - 1$$

$$\cos(2t + \pi) = -1 + 2 \sin^2(t)$$

$$-\cos(2t) = -(1 - 2 \sin^2(t))$$

$$\cos(2t) = \cos(2t)$$

Dit klopt voor elke t , dus bij K hoort de formule $y = \frac{1}{2}x^2 - 1$ met $-2 \leq x \leq 2$.

- 56**
- Substitutie van
- $x = \sin(t)$
- en
- $y = \sin(2t)$
- in
- $y^2 = 4x^2 - 4x^4$
- geeft

$$\sin^2(2t) = 4 \sin^2(t) - 4 \sin^4(t)$$

$$(2 \sin(t) \cos(t))^2 = 4 \sin^2(t) (1 - \sin^2(t))$$

$$4 \sin^2(t) \cos^2(t) = 4 \sin^2(t) \cos^2(t)$$

Dit klopt voor elke t .

$$\left. \begin{array}{l} x = \sin(t) \\ -1 \leq \sin(t) \leq 1 \end{array} \right\} \text{Bij de baan van } P \text{ hoort de formule } y^2 = 4x^2 - 4x^4 \text{ met } -1 \leq x \leq 1.$$

- 57 a**
- $-1 \leq \sin(t) \leq 1$
- , dus
- $-1 \leq x \leq 1$
- .

$$-1 \leq \sin(3t) \leq 1, \text{ dus } -1 \leq y \leq 1.$$

Dus de keerpunten van K zijn $(-1, 1)$ en $(1, -1)$.

- b**
- Substitutie van
- $x = \sin(t)$
- en
- $y = \sin(3t)$
- in
- $y = 3x - 4x^3$
- geeft

$$\sin(3t) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t + 2t) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t) \cos(2t) + \cos(t) \sin(2t) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t) (1 - 2 \sin^2(t)) + \cos(t) \cdot 2 \sin(t) \cos(t) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t) - 2 \sin^3(t) + 2 \sin(t) \cos^2(t) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t) - 2 \sin^3(t) + 2 \sin(t) \cdot (1 - \sin^2(t)) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t) - 2 \sin^3(t) + 2 \sin(t) - 2 \sin^3(t) = 3 \sin(t) - 4 \sin^3(t)$$

$$3 \sin(t) - 4 \sin^3(t) = 3 \sin(t) - 4 \sin^3(t)$$

Dit klopt voor elke t , dus alle punten van K liggen op de grafiek van $y = 3x - 4x^3$.

58 $-2 \leq 2 \cos(t) \leq 2$, dus $-2 \leq x \leq 2$ en dit geeft $c = -2$ en $d = 2$.

$$-1 \leq \cos(3t) \leq 1, \text{ dus } -1 \leq y \leq 1.$$

Dus de keerpunten zijn $(-2, -1)$ en $(2, 1)$.

$$x(\frac{1}{3}\pi) = 2 \cos(\frac{1}{3}\pi) = 2 \cdot \frac{1}{2} = 1 \text{ en } y(\frac{1}{3}\pi) = \cos(\pi) = -1, \text{ dus door } (1, -1).$$

$$\left. \begin{array}{l} y = ax^3 + bx \\ \text{door } (2, 1) \end{array} \right\} \begin{array}{l} a \cdot 2^3 + b \cdot 2 = 1 \\ 8a + 2b = 1 \end{array}$$

$$\left. \begin{array}{l} y = ax^3 + bx \\ \text{door } (1, -1) \end{array} \right\} \begin{array}{l} a \cdot 1^3 + b \cdot 1 = -1 \\ a + b = -1 \end{array}$$

$$\left\{ \begin{array}{l} 8a + 2b = 1 \\ a + b = -1 \end{array} \right| \begin{array}{l} 1 \\ 2 \end{array} \text{ geeft } \left\{ \begin{array}{l} 8a + 2b = 1 \\ 2a + 2b = -2 \end{array} \right. \quad \begin{array}{l} 6a = 3 \\ a = \frac{1}{2} \end{array}$$

$$\left. \begin{array}{l} a = \frac{1}{2} \\ a + b = -1 \end{array} \right\} \begin{array}{l} \frac{1}{2} + b = -1 \\ b = -1\frac{1}{2} \end{array}$$

Vermoedelijk hoort bij de baan van P de formule $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ met $-2 \leq x \leq 2$.

Substitutie van $x = 2 \cos(t)$ en $y = \cos(3t)$ in $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ geeft

$$\cos(3t) = \frac{1}{2}(2 \cos(t))^3 - 1\frac{1}{2} \cdot 2 \cos(t)$$

$$\cos(3t) = \frac{1}{2} \cdot 8 \cos^3(t) - 3 \cos(t)$$

$$\cos(2t) \cos(t) - \sin(2t) \sin(t) = 4 \cos^3(t) - 3 \cos(t)$$

$$(2 \cos^2(t) - 1) \cos(t) - 2 \sin(t) \cos(t) \sin(t) = 4 \cos^3(t) - 3 \cos(t)$$

$$2 \cos^3(t) - \cos(t) - 2 \sin^2(t) \cos(t) = 4 \cos^3(t) - 3 \cos(t)$$

$$2 \cos^3(t) - \cos(t) - 2(1 - \cos^2(t)) \cos(t) = 4 \cos^3(t) - 3 \cos(t)$$

$$2 \cos^3(t) - \cos(t) - 2 \cos(t) + 2 \cos^3(t) = 4 \cos^3(t) - 3 \cos(t)$$

$$4 \cos^3(t) - 3 \cos(t) = 4 \cos^3(t) - 3 \cos(t)$$

Dit klopt voor elke t , dus bij de baan van P hoort de formule $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ met $-2 \leq x \leq 2$.

59 Noem M het midden van OP .

Er geldt $MS \perp OP$ en $MS = OM = \frac{1}{2}OP$

$$\text{Dus } \vec{s} = \vec{m} + \vec{MS} = \frac{1}{2}\vec{p} + \vec{m}_L = \frac{1}{2}\vec{p} + \frac{1}{2}\vec{p}_L.$$

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60 $\vec{t} = \frac{1}{2}\vec{p} + \frac{1}{2}\vec{p}_R = \frac{1}{2} \begin{pmatrix} 2 \sin(t) \\ 2 \sin(2t) \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 2 \sin(2t) \\ -2 \sin(t) \end{pmatrix} = \begin{pmatrix} \sin(t) + \sin(2t) \\ \sin(2t) - \sin(t) \end{pmatrix}$

$$\text{Dus } \begin{cases} x_T(t) = \sin(t) + \sin(2t) \\ y_T(t) = \sin(2t) - \sin(t) \end{cases}$$

61 a $\vec{q} = \vec{a} + \vec{AQ} = \vec{a} + \frac{1}{2}\vec{AP} + \frac{1}{2}\vec{AP}_R$

$$\vec{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 2 \sin(t) \\ 2 \sin(2t) \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \sin(t) - 2 \\ 2 \sin(2t) \end{pmatrix}$$

$$\vec{AP}_R = \begin{pmatrix} 2 \sin(2t) \\ 2 - 2 \sin(t) \end{pmatrix}$$

$$\vec{q} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 2 \sin(t) - 2 \\ 2 \sin(2t) \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 2 \sin(2t) \\ 2 - 2 \sin(t) \end{pmatrix} = \begin{pmatrix} 2 + \sin(t) - 1 + \sin(2t) \\ \sin(2t) + 1 - \sin(t) \end{pmatrix} = \begin{pmatrix} \sin(2t) + \sin(t) + 1 \\ \sin(2t) - \sin(t) + 1 \end{pmatrix}$$

$$\text{Dus } \begin{cases} x_Q(t) = \sin(2t) + \sin(t) + 1 \\ y_Q(t) = \sin(2t) - \sin(t) + 1 \end{cases}$$

$$\mathbf{b} \quad \vec{r} = \vec{a} + \overrightarrow{AR} = \vec{a} + \overrightarrow{AP_R} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \sin(2t) \\ 2 - 2 \sin(t) \end{pmatrix} = \begin{pmatrix} 2 + 2 \sin(2t) \\ 2 - 2 \sin(t) \end{pmatrix}$$

$$y = 1 \text{ geeft } 2 - 2 \sin(t) = 1$$

$$-2 \sin(t) = -1$$

$$\sin(t) = \frac{1}{2}$$

$$t = \frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{5}{6}\pi + k \cdot 2\pi$$

$$t \text{ in } \langle 0, \pi \rangle \text{ geeft } t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi$$

$$x(\frac{1}{6}\pi) = 2 + 2 \sin(\frac{1}{3}\pi) = 2 + 2 \cdot \frac{1}{2}\sqrt{3} = 2 + \sqrt{3}$$

$$x(\frac{5}{6}\pi) = 2 + 2 \sin(\frac{5}{3}\pi) = 2 + 2 \cdot -\frac{1}{2}\sqrt{3} = 2 - \sqrt{3}$$

Dus de snijpunten zijn $(2 + \sqrt{3}, 1)$ en $(2 - \sqrt{3}, 1)$.

$$\mathbf{62} \quad \mathbf{a} \quad \vec{q} = \vec{p} + \overrightarrow{PQ} = \vec{p} + \overrightarrow{PA_L}$$

$$\overrightarrow{PA} = \vec{a} - \vec{p} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \sin(t) \\ 2 \sin(t - \frac{1}{4}\pi) \end{pmatrix} = \begin{pmatrix} 4 - 2 \sin(t) \\ -2 \sin(t - \frac{1}{4}\pi) \end{pmatrix}$$

$$\overrightarrow{PA_L} = \begin{pmatrix} 2 \sin(t - \frac{1}{4}\pi) \\ 4 - 2 \sin(t) \end{pmatrix}$$

$$\vec{q} = \begin{pmatrix} 2 \sin(t) \\ 2 \sin(t - \frac{1}{4}\pi) \end{pmatrix} + \begin{pmatrix} 2 \sin(t - \frac{1}{4}\pi) \\ 4 - 2 \sin(t) \end{pmatrix} = \begin{pmatrix} 2 \sin(t) + 2 \sin(t - \frac{1}{4}\pi) \\ 2 \sin(t - \frac{1}{4}\pi) + 4 - 2 \sin(t) \end{pmatrix}$$

$$\text{Dus } \begin{cases} x_Q(t) = 2 \sin(t) + 2 \sin(t - \frac{1}{4}\pi) \\ y_Q(t) = 2 \sin(t - \frac{1}{4}\pi) - 2 \sin(t) + 4 \end{cases}$$

$$\mathbf{b} \quad \text{Voer in } y_1 = 2 \sin(x - \frac{1}{4}\pi) - 2 \sin(x) + 4.$$

De optie minimum geeft $x = 0,392\dots$ en $y = 2,469\dots$

Het maximum van $y_P(t) = 2 \sin(t - \frac{1}{4}\pi)$ is 2.

Dus het laagste punt van de baan van Q ligt hoger dan het hoogste punt van de baan van P .

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$$\mathbf{63} \quad \mathbf{a} \quad \vec{s} = \vec{a} + \overrightarrow{AS} = \vec{a} + \frac{1}{2}\overrightarrow{AP} + \frac{1}{2}\overrightarrow{AP_R}$$

$$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 4 \cos(t) \\ 6 \sin(t) \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \cos(t) - 4 \\ 6 \sin(t) \end{pmatrix} \text{ geeft } \overrightarrow{AP_R} = \begin{pmatrix} 6 \sin(t) \\ 4 - 4 \cos(t) \end{pmatrix}$$

$$\vec{s} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 4 \cos(t) - 4 \\ 6 \sin(t) \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \sin(t) \\ 4 - 4 \cos(t) \end{pmatrix} = \begin{pmatrix} 4 + 2 \cos(t) - 2 + 3 \sin(t) \\ 3 \sin(t) + 2 - 2 \cos(t) \end{pmatrix} = \begin{pmatrix} 2 + 3 \sin(t) + 2 \cos(t) \\ 2 + 3 \sin(t) - 2 \cos(t) \end{pmatrix}$$

$$\text{Dus } \begin{cases} x_S(t) = 2 + 3 \sin(t) + 2 \cos(t) \\ y_S(t) = 2 + 3 \sin(t) - 2 \cos(t) \end{cases}$$

$$\mathbf{b} \quad \text{Substitueren van } x = 4 \cos(t) \text{ en } y = 6 \sin(t) \text{ in } 9x^2 + 4y^2 = 144 \text{ geeft}$$

$$9 \cdot (4 \cos(t))^2 + 4 \cdot (6 \sin(t))^2 = 144$$

$$9 \cdot 16 \cos^2(t) + 4 \cdot 36 \sin^2(t) = 144$$

$$144 \cos^2(t) + 144 \sin^2(t) = 144$$

$$\cos^2(t) + \sin^2(t) = 1.$$

Dit klopt voor elke t , dus bij de baan van P hoort de formule $9x^2 + 4y^2 = 144$.

$$\mathbf{c} \quad x = 2 + 3 \sin(t) + 2 \cos(t) \text{ en } y = 2 + 3 \sin(t) - 2 \cos(t) \text{ substitueren in } 9x^2 + 4y^2 = 144 \text{ geeft}$$

$$9(2 + 3 \sin(t) + 2 \cos(t))^2 + 4(2 + 3 \sin(t) - 2 \cos(t))^2 = 144.$$

$$\text{Voer in } y_1 = 9(2 + 3 \sin(x) + 2 \cos(x))^2 + 4(2 + 3 \sin(x) - 2 \cos(x))^2 \text{ en } y_2 = 144.$$

De optie snijpunt geeft $x = 2,583\dots$

$$t = 2,583\dots \text{ geeft } x = 2 + 3 \sin(2,583\dots) + 2 \cos(2,583\dots) \approx 1,89 \text{ en}$$

$$y = 2 + 3 \sin(2,583\dots) - 2 \cos(2,583\dots) \approx 5,29.$$

Dus $B(1,89; 5,29)$.

$$64 \quad \vec{q} = \frac{1}{2}\vec{p} + \frac{1}{2}\vec{p}_R = \frac{1}{2} \begin{pmatrix} 2 \sin(2t) \\ 2 \sin(t + \frac{1}{4}\pi) \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 2 \sin(t + \frac{1}{4}\pi) \\ -2 \sin(2t) \end{pmatrix} = \begin{pmatrix} \sin(2t) + \sin(t + \frac{1}{4}\pi) \\ \sin(t + \frac{1}{4}\pi) - \sin(2t) \end{pmatrix}$$

$$\text{Dus } \begin{cases} x_Q(t) = \sin(2t) + \sin(t + \frac{1}{4}\pi) \\ y_Q(t) = \sin(t + \frac{1}{4}\pi) - \sin(2t) \end{cases}$$

$x = \sin(2t) + \sin(t + \frac{1}{4}\pi)$ en $y = \sin(t + \frac{1}{4}\pi) - \sin(2t)$ substitueren in $y = -x$ geeft

$$\sin(t + \frac{1}{4}\pi) - \sin(2t) = -\sin(2t) - \sin(t + \frac{1}{4}\pi)$$

$$2 \sin(t + \frac{1}{4}\pi) = 0$$

$$\sin(t + \frac{1}{4}\pi) = 0$$

$$t + \frac{1}{4}\pi = k \cdot \pi$$

$$t = -\frac{1}{4}\pi + k \cdot \pi$$

$$\frac{1}{4}\pi \leq t \leq 1\frac{1}{4}\pi \text{ geeft } t = \frac{3}{4}\pi$$

$$t = \frac{3}{4}\pi \text{ geeft } x = \sin(\frac{1}{2}\pi) + \sin(\pi) = -1$$

Dus $A(-1, 1)$.

$$65 \quad x = 2 \sin(2t) \text{ en } y = 2 \sin(t + \frac{1}{4}\pi) \text{ substitueren in } y = -x \text{ geeft}$$

$$2 \sin(t + \frac{1}{4}\pi) = -2 \sin(2t)$$

$$\sin(t + \frac{1}{4}\pi) = -\sin(2t)$$

$$\sin(t + \frac{1}{4}\pi) = \sin(2t + \pi)$$

$$t + \frac{1}{4}\pi = 2t + \pi + k \cdot 2\pi \vee t + \frac{1}{4}\pi = \pi - (2t + \pi) + k \cdot 2\pi$$

$$-t = \frac{3}{4}\pi + k \cdot 2\pi \vee t + \frac{1}{4}\pi = \pi - 2t - \pi + k \cdot 2\pi$$

$$t = -\frac{3}{4}\pi + k \cdot 2\pi \vee 3t = -\frac{1}{4}\pi + k \cdot 2\pi$$

$$t = -\frac{3}{4}\pi + k \cdot 2\pi \vee t = -\frac{1}{12}\pi + k \cdot \frac{2}{3}\pi$$

$$\frac{1}{4}\pi \leq t \leq 1\frac{1}{4}\pi \text{ geeft } t = \frac{7}{12}\pi \vee t = 1\frac{1}{4}\pi$$

$$x_P(\frac{7}{12}\pi) = 2 \sin(\frac{1}{6}\pi) = 2 \cdot \frac{1}{2} = 1 \text{ en } y_P(\frac{7}{12}\pi) = 2 \sin(\frac{5}{6}\pi) = 2 \cdot \frac{1}{2} = 1$$

Dus het punt $A(-1, 1)$ ligt op beide banen.

$$x_P(t) = 2 \sin(2t) \text{ geeft } x_P'(t) = 4 \cos(2t)$$

$$y_P(t) = 2 \sin(t + \frac{1}{4}\pi) \text{ geeft } y_P'(t) = 2 \cos(t + \frac{1}{4}\pi)$$

$$\vec{v}_P(\frac{7}{12}\pi) = \begin{pmatrix} x_P'(\frac{7}{12}\pi) \\ y_P'(\frac{7}{12}\pi) \end{pmatrix} = \begin{pmatrix} 4 \cos(\frac{1}{6}\pi) \\ 2 \cos(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} 4 \cdot \frac{1}{2}\sqrt{3} \\ 2 \cdot -\frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} 2\sqrt{3} \\ -\sqrt{3} \end{pmatrix}$$

$$x_Q(t) = \sin(2t) + \sin(t + \frac{1}{4}\pi) \text{ geeft } x_Q'(t) = 2 \cos(2t) + \cos(t + \frac{1}{4}\pi)$$

$$y_Q(t) = \sin(t + \frac{1}{4}\pi) - \sin(2t) \text{ geeft } y_Q'(t) = \cos(t + \frac{1}{4}\pi) - 2 \cos(2t)$$

$$\vec{v}_Q(\frac{3}{4}\pi) = \begin{pmatrix} x_Q'(\frac{3}{4}\pi) \\ y_Q'(\frac{3}{4}\pi) \end{pmatrix} = \begin{pmatrix} 2 \cos(\frac{1}{2}\pi) + \cos(\pi) \\ \cos(\pi) - 2 \cos(\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} 2 \cdot 0 + -1 \\ -1 - 2 \cdot 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$\begin{pmatrix} -2\sqrt{3} \\ -\sqrt{3} \end{pmatrix}$ is niet evenwijdig met $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$, dus de banen raken elkaar niet op de lijn $y = -x$.

Diagnostische toets

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$$1 \quad a \quad \sin(3x - \frac{1}{4}\pi) = \cos(2x)$$

$$\cos(3x - \frac{3}{4}\pi) = \cos(2x)$$

$$3x - \frac{3}{4}\pi = 2x + k \cdot 2\pi \vee 3x - \frac{3}{4}\pi = -2x + k \cdot 2\pi$$

$$x = \frac{3}{4}\pi + k \cdot 2\pi \vee 5x = \frac{3}{4}\pi + k \cdot 2\pi$$

$$x = \frac{3}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{20}\pi + k \cdot \frac{2}{5}\pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{3}{4}\pi \vee x = \frac{3}{20}\pi \vee x = \frac{11}{20}\pi \vee x = \frac{19}{20}\pi$$

b $2 \cos^2(2x) + \sin(2x) = 1$
 $2(1 - \sin^2(2x)) + \sin(2x) = 1$
 $2 - 2 \sin^2(2x) + \sin(2x) = 1$
 $2 \sin^2(2x) - \sin(2x) - 1 = 0$
 Stel $\sin(2x) = u$.
 $2u^2 - u - 1 = 0$
 $D = (-1)^2 - 4 \cdot 2 \cdot -1 = 9$
 $u = \frac{1+3}{4} = 1 \vee u = \frac{1-3}{4} = -\frac{1}{2}$
 $\sin(2x) = 1 \vee \sin(2x) = -\frac{1}{2}$
 $2x = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{4}\pi + k \cdot \pi \vee x = -\frac{1}{12}\pi + k \cdot \pi \vee x = \frac{7}{12}\pi + k \cdot \pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{4}\pi \vee x = \frac{1}{4}\pi \vee x = \frac{11}{12}\pi \vee x = \frac{11}{12}\pi \vee x = \frac{7}{12}\pi \vee x = \frac{7}{12}\pi$

c $\cos(\frac{2}{3}\pi t) = -\sin(\frac{1}{6}\pi t)$
 $\cos(\frac{2}{3}\pi t) = \sin(\frac{1}{6}\pi t + \pi)$
 $\cos(\frac{2}{3}\pi t) = \cos(\frac{1}{6}\pi t + \frac{1}{2}\pi)$
 $\frac{2}{3}\pi t = \frac{1}{6}\pi t + \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{2}{3}\pi t = -\frac{1}{6}\pi t - \frac{1}{2}\pi + k \cdot 2\pi$
 $\frac{7}{30}\pi t = \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{7}{30}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi$
 $t = 2\frac{1}{7} + k \cdot 8\frac{4}{7} \vee t = -\frac{15}{17} + k \cdot 3\frac{9}{17}$
 $t \text{ in } [0, 10] \text{ geeft } t = 2\frac{1}{7} \vee t = 2\frac{11}{17} \vee t = 6\frac{3}{17} \vee t = 9\frac{12}{17}$

2 a $\sin(\frac{1}{2}x) \cos(\frac{1}{2}x) + \cos(\frac{1}{2}x) \sin(\frac{1}{2}x) = \sin(\frac{1}{2}x + \frac{1}{2}x) = \sin(x)$
b $\sin(3x) \sin(5x) + \cos(3x) \cos(5x) = \cos(3x - 5x) = \cos(-2x) = \cos(2x)$
c $2 \sin(3x) \cos(3x) = \sin(2 \cdot 3x) = \sin(6x)$
d $2 \cos^2(5x) - 1 = \cos(2 \cdot 5x) = \cos(10x)$

3 a $\sin(x + \frac{1}{3}\pi) = 2 \sin(2x) \cos(2x)$
 $\sin(x + \frac{1}{3}\pi) = \sin(4x)$
 $x + \frac{1}{3}\pi = 4x + k \cdot 2\pi \vee x + \frac{1}{3}\pi = \pi - 4x + k \cdot 2\pi$
 $-3x = -\frac{1}{3}\pi + k \cdot 2\pi \vee 5x = \frac{2}{3}\pi + k \cdot 2\pi$
 $x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{2}{15}\pi + k \cdot \frac{2}{5}\pi$
b $\sin^2(2x) + \frac{1}{4} = \cos(4x)$
 $\sin^2(2x) + \frac{1}{4} = 1 - 2 \sin^2(2x)$
 $3 \sin^2(2x) = \frac{3}{4}$
 $\sin^2(2x) = \frac{1}{4}$
 $\sin(2x) = \frac{1}{2} \vee \sin(2x) = -\frac{1}{2}$
 $2x = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \frac{5}{6}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \frac{11}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{12}\pi + k \cdot \pi \vee x = \frac{5}{12}\pi + k \cdot \pi \vee x = -\frac{1}{12}\pi + k \cdot \pi \vee x = \frac{7}{12}\pi + k \cdot \pi$

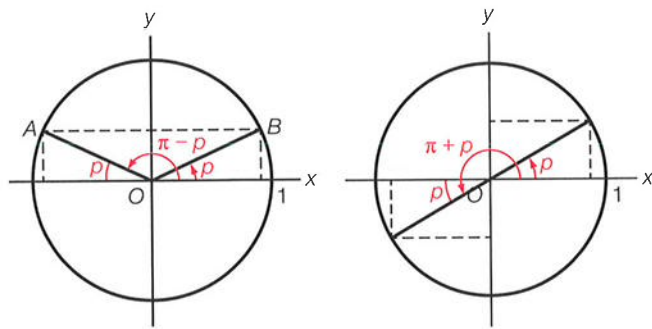
4 $y = \sin^2(2x) + 2 \cos^2(2x) + 3 \cos(4x) = \sin^2(2x) + \cos^2(2x) + \cos^2(2x) + 3(2 \cos^2(2x) - 1)$
 $= 1 + \cos^2(2x) + 6 \cos^2(2x) - 3 = -2 + 7 \cos^2(2x) = -2 + 7 \cos^2(x) - 3\frac{1}{2} + 3\frac{1}{2}$
 $= 1\frac{1}{2} + 3\frac{1}{2}(2 \cos^2(x) - 1) = 1\frac{1}{2} + 3\frac{1}{2} \cos(4x)$

5 a $f(\frac{1}{2}\pi - p) + f(\frac{1}{2}\pi + p) = \sin(3\pi - 2p) + \cos(\frac{1}{2}\pi - p) + \sin(3\pi + 2p) + \cos(\frac{1}{2}\pi + p)$
 $= \sin(3\pi) \cos(2p) - \cos(3\pi) \sin(2p) + \cos(\frac{1}{2}\pi) \cos(p) + \sin(\frac{1}{2}\pi) \sin(p)$
 $+ \sin(3\pi) \cos(2p) + \cos(3\pi) \sin(2p) + \cos(\frac{1}{2}\pi) \cos(p) - \sin(\frac{1}{2}\pi) \sin(p)$
 $= 2 \sin(3\pi) \cos(2p) + 2 \cos(\frac{1}{2}\pi) \cos(p) = 2 \cdot 0 \cdot \cos(2p) + 2 \cdot 0 \cdot \cos(p) = 0$

Er geldt $f(\frac{1}{2}\pi - p) + f(\frac{1}{2}\pi + p) = 0$.

Dus de grafiek van f is symmetrisch in het punt $(\frac{1}{2}\pi, 0)$.

b



$$f(\pi - p) = 2 \sin^2(\pi - p) \cos(\pi - p) = 2 \sin^2(p) \cdot -\cos(p) = -2 \sin^2(p) \cos(p)$$

$$f(\pi + p) = 2 \sin^2(\pi + p) \cos(\pi + p) = 2(-\sin(p))^2 \cdot -\cos(p) = -2 \sin^2(p) \cos(p)$$

Er geldt $f(\pi - p) = f(\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \pi$.

6 $f(x) = 0$ geeft $2 \sin(x) = 0$

$$\sin(x) = 0$$

$$x = k \cdot \pi$$

$$\begin{aligned} I(L) &= \pi \int_0^\pi (2 \sin(x))^2 dx = \pi \int_0^\pi 4 \sin^2(x) dx = \pi \int_0^\pi 2(1 - \cos(2x)) dx = \pi \int_0^\pi (2 - 2 \cos(2x)) dx \\ &= \pi [2x - \sin(2x)]_0^\pi = \pi(2\pi - 0 - 0) = 2\pi^2 \end{aligned}$$

7 $f(x) = 0$ geeft $1 - 2 \cos^2(x) - \cos(x) = 0$

$$\text{Stel } \cos(x) = u.$$

$$1 - 2u^2 - u = 0$$

$$2u^2 + u - 1 = 0$$

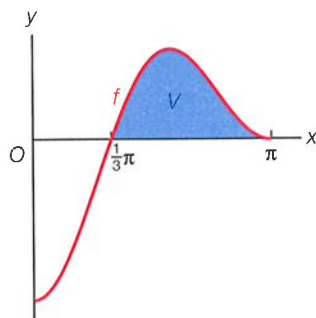
$$D = 1^2 - 4 \cdot 2 \cdot -1 = 9$$

$$u = \frac{-1 + 3}{4} = \frac{1}{2} \vee u = \frac{-1 - 3}{4} = -1$$

$$\cos(x) = \frac{1}{2} \vee \cos(x) = -1$$

$$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi \vee x = \pi + k \cdot 2\pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{1}{3}\pi \vee x = \pi$$



$$\begin{aligned} O(V) &= \int_{\frac{1}{3}\pi}^\pi (1 - 2 \cos^2(x) - \cos(x)) dx = \int_{\frac{1}{3}\pi}^\pi (-\cos(2x) - \cos(x)) dx \\ &= \left[-\frac{1}{2} \sin(2x) - \sin(x) \right]_{\frac{1}{3}\pi}^\pi = -\frac{1}{2} \sin(2\pi) - \sin(\pi) - \left(-\frac{1}{2} \sin\left(\frac{2}{3}\pi\right) - \sin\left(\frac{1}{3}\pi\right) \right) \\ &= 0 - 0 + \frac{1}{2} \cdot \frac{1}{2} \sqrt{3} + \frac{1}{2} \sqrt{3} = \frac{3}{4} \sqrt{3} \end{aligned}$$

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- 8 a** Het punt A doorloopt de cirkel met middelpunt O en straal 2.
 Het punt B doorloopt de cirkel met middelpunt O en straal 3.
 De minimale afstand is $3 - 2 = 1$ voor $\frac{1}{2}t = 2t + k \cdot 2\pi$

$$-1\frac{1}{2}t = k \cdot 2\pi$$

$$t = k \cdot 1\frac{1}{3}\pi$$

Dus de afstand is minimaal voor $t = 0$, $t = 1\frac{1}{3}\pi$, $t = 2\frac{2}{3}\pi$ en $t = 4\pi$.

De maximale afstand is $3 + 2 = 5$ voor $\frac{1}{2}t = 2t + \pi + k \cdot 2\pi$

$$-1\frac{1}{2}t = \pi + k \cdot 2\pi$$

$$t = -\frac{2}{3}\pi + k \cdot 1\frac{1}{3}\pi$$

Dus de afstand is maximaal voor $t = \frac{2}{3}\pi$, $t = 2\pi$ en $t = 3\frac{1}{3}\pi$.

- b** $AB^2 = (3 \cos(2t) - 2 \cos(\frac{1}{2}t))^2 + (3 \sin(2t) - 2 \sin(\frac{1}{2}t))^2$
 $= 9 \cos^2(2t) - 12 \cos(2t) \cos(\frac{1}{2}t) + 4 \cos^2(\frac{1}{2}t) + 9 \sin^2(2t) - 12 \sin(2t) \sin(\frac{1}{2}t) + 4 \sin^2(\frac{1}{2}t)$
 $= 9 + 4 - 12(\cos(2t) \cos(\frac{1}{2}t) + \sin(2t) \sin(\frac{1}{2}t))$
 $= 13 - 12 \cos(2t - \frac{1}{2}t)$
 $= 13 - 12 \cos(\frac{3}{2}t)$

$$\text{Dus } AB = \sqrt{13 - 12 \cos(\frac{3}{2}t)}.$$

- c** $AB = \sqrt{7}$ geeft $\sqrt{13 - 12 \cos(\frac{3}{2}t)} = \sqrt{7}$
 $13 - 12 \cos(\frac{3}{2}t) = 7$
 $12 \cos(\frac{3}{2}t) = 6$
 $\cos(\frac{3}{2}t) = \frac{1}{2}$
 $\frac{3}{2}t = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{3}{2}t = -\frac{1}{3}\pi + k \cdot 2\pi$
 $t = \frac{2}{9}\pi + k \cdot 1\frac{1}{3}\pi \vee t = -\frac{2}{9}\pi + k \cdot 1\frac{1}{3}\pi$

t in $[0, 4\pi]$ geeft $t = \frac{2}{9}\pi \vee t = 1\frac{5}{9}\pi \vee t = 2\frac{8}{9}\pi \vee t = 1\frac{1}{9}\pi \vee t = 2\frac{4}{9}\pi \vee t = 3\frac{7}{9}\pi$

- 9 a** $u = b \sin(2\pi ft)$
 $b = \text{amplitude} = 5$
 $f = 4 \text{ Hz}$
 Dus $u = 5 \sin(8\pi t)$.

- b** Per trilling is de afgelegde afstand $4 \cdot 5 = 20 \text{ dm} = 2 \text{ m}$.
 Er zijn 4 trillingen per seconde, dus per uur zijn er $3600 \cdot 4 = 14400$ trillingen.
 Dus in een uur is de afgelegde afstand $14400 \cdot 2 \text{ m} = 28800 \text{ m} = 28,8 \text{ km}$.

- 10 a** $y = -\frac{1}{2}\sqrt{3}$ geeft $\cos(2t - \frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$
 $2t - \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee 2t - \frac{1}{3}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$
 $2t = 1\frac{1}{6}\pi + k \cdot 2\pi \vee 2t = -\frac{1}{2}\pi + k \cdot 2\pi$
 $t = \frac{7}{12}\pi + k \cdot \pi \vee t = -\frac{1}{4}\pi + k \cdot \pi$

t in $[0, 1\frac{1}{2}\pi]$ geeft $t = \frac{7}{12}\pi \vee t = \frac{3}{4}\pi$

$$x(\frac{7}{12}\pi) = \sin(1\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}$$

$$x(\frac{3}{4}\pi) = \sin(2\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2}$$

$$\text{Dus } AB = \frac{1}{2}\sqrt{2} - (-\frac{1}{2}\sqrt{2}) = \sqrt{2}.$$

- b** $-1 \leq \sin(3t) \leq 1$, dus $-1 \leq x \leq 1$.
 $-1 \leq \cos(2t - \frac{1}{3}\pi) \leq 1$, dus $-1 \leq y \leq 1$.
 Dus de keerpunten zijn $(-1, 1)$ en $(1, 1)$.

c $x = 0$ geeft $\sin(3t) = 0$

$$3t = k \cdot \pi$$

$$t = k \cdot \frac{1}{3}\pi$$

x in $[0, 1\frac{1}{2}\pi]$ geeft $t = 0 \vee t = \frac{1}{3}\pi \vee t = \frac{2}{3}\pi \vee t = \pi \vee t = 1\frac{1}{3}\pi$

$$y(0) = \cos(-\frac{1}{3}\pi) = \frac{1}{2}$$

$$y(\frac{1}{3}\pi) = \cos(\frac{1}{3}\pi) = \frac{1}{2}$$

$$y(\frac{2}{3}\pi) = \cos(\pi) = -1$$

$$y(\pi) = \cos(1\frac{2}{3}\pi) = \frac{1}{2}$$

$$y(1\frac{1}{3}\pi) = \cos(2\frac{1}{3}\pi) = \frac{1}{2}$$

$$x(t) = \sin(3t) \text{ geeft } x'(t) = 3 \cos(3t)$$

$$y(t) = \cos(2t - \frac{1}{3}\pi) \text{ geeft } y'(t) = -2 \sin(2t - \frac{1}{3}\pi)$$

$$\vec{v}(\frac{1}{3}\pi) = \begin{pmatrix} x'(\frac{1}{3}\pi) \\ y'(\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 3 \cos(\pi) \\ -2 \sin(\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 3 \cdot -1 \\ -2 \cdot \frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} -3 \\ -\sqrt{3} \end{pmatrix} \text{ en}$$

$$\vec{v}(\pi) = \begin{pmatrix} x'(\pi) \\ y'(\pi) \end{pmatrix} = \begin{pmatrix} 3 \cos(3\pi) \\ -2 \sin(1\frac{2}{3}\pi) \end{pmatrix} = \begin{pmatrix} 3 \cdot -1 \\ -2 \cdot -\frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix}$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} -3 \\ -\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix} \right|}{\left| \begin{pmatrix} -3 \\ -\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix} \right|} = \frac{|9 - 3|}{\sqrt{(-3)^2 + (-\sqrt{3})^2} \cdot \sqrt{(-3)^2 + (\sqrt{3})^2}} = \frac{6}{\sqrt{12} \cdot \sqrt{12}} = \frac{6}{12} = \frac{1}{2}$$

Dus $\varphi = 60^\circ$.

$$\begin{aligned} \text{d } v(\frac{2}{3}\pi) &= \sqrt{(x'(\frac{2}{3}\pi))^2 + (y'(\frac{2}{3}\pi))^2} = \sqrt{(3 \cos(2\pi))^2 + (-2 \sin(\pi))^2} = \sqrt{(3 \cdot 1)^2 + (-2 \cdot 0)^2} \\ &= \sqrt{3^2 + 0^2} = 3 \end{aligned}$$

Dus de baansnelheid is 3.

$$\text{e } \vec{q} = \overrightarrow{OP} + \overrightarrow{PQ} = \vec{p} + \vec{p}_L = \begin{pmatrix} \sin(3t) \\ \cos(2t - \frac{1}{3}\pi) \end{pmatrix} + \begin{pmatrix} -\cos(2t - \frac{1}{3}\pi) \\ \sin(3t) \end{pmatrix} = \begin{pmatrix} \sin(3t) - \cos(2t - \frac{1}{3}\pi) \\ \cos(2t - \frac{1}{3}\pi) + \sin(3t) \end{pmatrix}$$

$$\text{Dus } \begin{cases} x_Q(t) = \sin(3t) - \cos(2t - \frac{1}{3}\pi) \\ y_Q(t) = \cos(2t - \frac{1}{3}\pi) + \sin(3t) \end{cases}$$

$$y = 0 \text{ geeft } \cos(2t - \frac{1}{3}\pi) + \sin(3t) = 0$$

$$\cos(2t - \frac{1}{3}\pi) = -\sin(3t)$$

$$\cos(2t - \frac{1}{3}\pi) = -\cos(3t - \frac{1}{2}\pi)$$

$$\cos(2t - \frac{1}{3}\pi) = \cos(3t + \frac{1}{2}\pi)$$

$$2t - \frac{1}{3}\pi = 3t + \frac{1}{2}\pi + k \cdot 2\pi \vee 2t - \frac{1}{3}\pi = -3t - \frac{1}{2}\pi + k \cdot 2\pi$$

$$-t = \frac{5}{6}\pi + k \cdot 2\pi \vee 5t = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{5}{6}\pi + k \cdot 2\pi \vee t = -\frac{1}{30}\pi + k \cdot \frac{2}{5}\pi$$

$$t \text{ in } [0, 1\frac{1}{2}\pi] \text{ geeft } t = 1\frac{1}{6}\pi \vee t = \frac{11}{30}\pi \vee t = \frac{23}{30}\pi$$

$$x_Q(1\frac{1}{6}\pi) = -2$$

$$x_Q(\frac{11}{30}\pi) \approx -0,618$$

$$x_Q(\frac{23}{30}\pi) \approx 1,618$$

Dus voor $t = \frac{23}{30}\pi$.